Finite memory quadratic Volterra model for the response prediction of a slender marine structure under a Morison load

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**A B S T R A C T**

A quadratic Volterra model with a finite nonlinear memory effect was introduced and applied to the time series prediction of a slender marine structure exposed to the Morison load. First, the unknown nonlinear single-input–single-output dynamic system was identified using the nonlinear autoregressive with exogenous input (NARX) technique based on the prepared datasets of the wave elevation and system response, which was obtained by running nonlinear time domain analysis for a certain short term sea state. The structure of NARX was designed in such a way that the linear part had infinite memory, whereas the nonlinear part had finite memory of a certain length. Second, the frequency domain Volterra kernels, both linear and quadratic, were derived analytically by applying the harmonic probing method to the identified system. To derive the frequency response functions, the sigmoidal function used in NARX to realize the nonlinear relationship between the input and output was expanded to polynomials based on the Taylor series expansion, so that the harmonics of same frequencies were easily matched between the input and output. Finally, the time series of the system response under arbitrarily given short term sea states were predicted using the quadratic Volterra series. The proposed methodology was used to predict the nonlinear dynamic response of a 2-dimensional free standing catenary riser exposed to a random ocean wave load, and the comparison between the prediction and simulation results was made on the probability distribution of the maximum excursion of riser top. The results show that the proposed methodology can successfully capture the nonlinear effects of the dynamic response of a slender marine structure induced by the quadratic term of the Morison formula.

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1. Introduction

Owing to the depletion of the fossil fuels in continental shelf reserves, deep sea oil and gas exploration is increasing so that the use of floating type offshore structures connected to the risers and mooring lines is widespread. These slender marine structures, in addition to the floater connected to it, are exposed to random ocean wave loads, and this randomly acting environmental load is one of the most important key design parameters guaranteeing the structural reliability of offshore facilities. The wave loads acting on the slender marine structure are normally assumed to be given in the form of the well-known Morison formula, which is composed of dynamically acting fluid inertia and drag forces. The inertia force is normally assumed to be proportional to the acceleration of the fluid particle and the drag force proportional to the square
of the relative velocity of the fluid particle and dynamically moving structure. Even under linear gravity wave theory, the drag force term in the Morison formula plays a role as a source of nonlinearity, which necessitates nonlinear time domain analysis to calculate the dynamic response of the structure under consideration. In terms of the nonlinearity source, the structural response sometimes participates. As far as the dynamic response of the slender offshore structure is concerned, the difference frequency components drive the structure to respond in an oscillatory fashion with relatively large magnitude. Subsequently, this large magnitude oscillatory motion of low frequency naturally induces additional nonlinearity in the structural part of the system, i.e., large displacement. Therefore, the nonlinearities of both the wave excitation and structural behavior have forced designers to rely on the computationally expensive nonlinear time domain finite element analysis to predict the structural response. The main difficulties associated with this approach, particularly in the early design stage, are the large number of load cases that need to be analyzed. Owing to their stochastic nature, ocean waves are usually characterized with hundreds of short-term wave spectra, each of which covers an approximately 3 h sea state, and all these individual short-term sea states need to be considered as load cases to perform long-term analysis. Therefore, direct nonlinear time domain finite element analysis with a large displacement normally takes several days to complete long-term analysis, which is one of the bottlenecks in the design process.

To overcome the above mentioned difficulties, many studies have focused on developing a less time consuming method to predict the long-term distribution of extreme response of offshore structures (Mazaheri and Downie, 2004; Moan et al., 2007; Rodrigues et al., 2007; Vazquez-Hernandez et al., 2011; Yasseri et al., 2010). Low and Langley (2008) proposed a hybrid time/frequency domain method to make accurate predictions of the response of coupled floater–mooring–riser system. In this method, the low frequency response of the system is simulated in the time domain, whereas the wave frequency response in frequency domain. Low (2011) extended this method to the fatigue analysis of moorings and risers, and showed that the calculated fatigue damage matches well with the results from the full time domain analysis. Passano and Maincon (2011) proposed an efficient and unbiased estimation method to predict the long-term extreme response distribution of a catenary riser. They introduced a nonlinear response predictor to estimate the response of the structure in all sea states based on the nonlinear time domain simulation for the selected relevant sea states. Najafian (2007) proposed a finite memory nonlinear system model to predict the dynamic response of a jacket structure exposed to a Morison load, and applied the method to three sample problems of a jacket structure with different natural frequencies. Gobat and Grosenbaugh (2001) proposed an empirical model for the dynamic tension caused by vertical motion at the top of a catenary mooring line. Their model calculated the standard deviation of the tension as a sum of an inertial term proportional to the heave acceleration and a drag term proportional to the quadratic heave velocity. On the other hand, the approach was limited to the wave frequency response only. Pascoal et al. (2005) proposed a simplified model to predict the motion behavior of a mooring line, which allows rapid coupled analysis of hull and mooring systems. They determined the parameters of the equivalent mooring line using an identification procedure where a cable dynamics model is excited by a random signal. The other branches of the efficient analysis on the slender marine structure focus mainly on the use of a system identification approach based on the artificial neural network technique. Pina et al. (2013) proposed a new surrogate model based on the artificial neural network to predict the force developing in the mooring lines and risers. They used a Nonlinear Autoregressive with Exogenous Input (NARX) scheme to identify the coupled system, and predicted the top tension of the riser under the prescribed motion of the floater. Pina et al. (2014a,b) also extended their work by applying a wavelet network to improve the accuracy. Christiansen et al. (2013) used a similar approach to predict the time series of the mooring line top tension without relying on the direct nonlinear time domain FE analysis and reported good correspondence between the predicted and simulated results.

The system identification approach may be coupled to a higher order Volterra model due to the inherent similarity between the two. A combination of the artificial neural network, particularly Nonlinear Autoregressive with Exogeneous Input (NARX) or Time Delayed Neural Network (TDNN), and harmonic probing method were studied by some researchers to identify the unknown nonlinear system by mapping the input to the output of the system. Wray and Green (1994) and Marmarelis and Zhao (1997) showed that the nonlinear Volterra kernels could be matched with the weights of the TDNN and derived the closed form relationship between the Volterra kernels and network weights. Worden et al. (1994) applied the Nonlinear Autoregressive Moving Average with Exogeneous Input (NARMAX) model to identify the nonlinear system of floating structure exposed to the wave loading. The unknown system was identified using the NARMAX method based on the vast amount of experimental data in a model basin. In addition, a full scale measurement data and nonlinear frequency response function up to the second order was obtained using the harmonic probing method. Chance et al. (1998) extended the correspondence of the Volterra kernels and TDNN to the NARX, where output of the system was fed back into the network as an additional input. They identified the nonlinear system using the NARX neural network model and linked it to the frequency response functions of the system using the harmonic probing method. They also examined the performance of different activation functions, such as hyperbolic tangent and polynomial functions. The proposed model was validated through an analysis of the Duffing type nonlinear oscillator with quadratic and cubic stiffness. Kim and Lee (2014) combined the NARX method with the Volterra series to predict the dynamic response of a slender tower structure exposed to random ocean waves.

The present study addresses an efficient system identification approach through which the response of a slender marine structure subjected to a Morison load can be predicted without relying on the time consuming direct analysis. The methodology was motivated by the fact that the memory effect of the nonlinear difference frequency term of the Morison load is finite. This simplifies the system identification process significantly because the recursive relationship of the system
response is limited to the linear part, whereas the nonlinearity is modeled with the exogenous input only. Once the system is identified, the linear and quadratic transfer functions were derived using the harmonic probing method, and the system response under an arbitrary short term sea state was predicted by the quadratic Volterra series.

2. Theoretical background

2.1. Volterra model

A Volterra series is used broadly in modeling the nonlinear response of a dynamic system, even though it is limited to the weakly nonlinear case. The basic concept of the Volterra series is similar to the Taylor series but the memory effect reflected in the Volterra series is what distinguishes it from the Taylor series. The reason why it is limited to the weakly nonlinear problem is that the Volterra series is of little value once too many higher order terms are included, which is practically difficult to handle. In general, the response of a system, \(y(t)\), under external excitation, \(x(t)\), can be expressed in terms of the Volterra series as in Eq. (1).

\[
y(t) = \sum_{n=1}^{N} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(t_1, t_2, \ldots, t_n) \prod_{j=1}^{n} x(t-j) dt_j,
\]

where \(h_n\) is the \(n\)th order Volterra kernel. When \(N=2\), Eq. (1) becomes the second order Volterra series, as expressed in Eq. (2).

\[
y(t) = \int_{-\infty}^{\infty} h_1(t_1)x(t-t_1)dt_1 + \int_{-\infty}^{\infty} h_2(t_1, t_2)x(t-t_1)x(t-t_2) dt_1 dt_2.
\]

The first term of Eq. (2) is the well-known convolution integral of the external excitation and the impulse response function of the system. The impulse response function is the unique nature of any dynamic system and becomes the first order frequency response function of a system when transformed to the frequency domain. The second term is the double convolution integral of the second order generalized impulse response function and external excitations at two different time instances, which is a multi-dimensional generalization of the linear case.

The linear transfer function (LTF) and quadratic transfer function (QTF) of a system can be obtained by taking Fourier transform on the Volterra kernel of each order. In the case of second order Volterra model, LTF and QTF are given as

\[
H_1(\omega_1) = \int_{-\infty}^{\infty} h_1(t_1)e^{-i\omega_1 t_1} dt_1
\]

\[
H_2(\omega_1, \omega_2) = \int_{-\infty}^{\infty} h_2(t_1, t_2)e^{-i\omega_1 t_1 - i\omega_2 t_2} dt_1 dt_2.
\]

QTF is symmetrical so that \(H_2(\omega_1, \omega_2) = H_2(\omega_2, \omega_1)\). In addition, \(H_2(\omega_1, \omega_2) = H_2^\ast(-\omega_1, -\omega_2)\), where the asterisk denotes a complex conjugate.

Once the Volterra kernel or frequency response function of each order is known, the time history of the response of a system that is subjected to the arbitrary excitation can be predicted without much computational burden. When the frequency response function up to the second order are known, the time history of the system response of both the first and second order, \(y_1(t), y_2(t)\), under bichromatic harmonic excitation can be expressed as

\[
y_1(t) = \sum_{j=1}^{3} A_j Re\left[H_1(\omega_j)e^{i(\omega_j t - \phi_j)}\right],
\]

\[
y_2(t) = \sum_{j=1}^{2} \sum_{k=1}^{2} A_j A_k Re\left[H_2(\omega_j, \omega_k)e^{i\left((\omega_j + \omega_k)t - (\phi_j + \phi_k)\right)}\right]
+ \sum_{j=1}^{2} \sum_{k=1}^{2} A_j A_k Re\left[H_2(\omega_j, -\omega_k)e^{i\left((\omega_j - \omega_k)t - (\phi_j - \phi_k)\right)}\right],
\]

where \(A_j, \omega_j\) and \(\phi_j\) mean the excitation magnitude, its circular frequency and phase angle, respectively.

As shown in Eq. (4), both the sum and difference frequency components comprise the second order response. The sum frequency component consists of \(2\omega_1\) and \(2\omega_2\) of two harmonic excitations of the same frequency together with \(\omega_k + \omega_l\) of two harmonic excitations of difference frequencies. The difference frequency components consists of a zero frequency term, i.e., mean value, originating from the two harmonic excitations of same frequency along with \(\omega_k - \omega_l\) of two harmonic excitations of difference frequencies. Owing to the difficulties associated with the direct estimation of Volterra kernel in time domain (Peyton Jones and Billings, 1990), the present study focuses on the frequency domain representation of Volterra kernel, i.e., higher order frequency response functions (HFRFs), which are sought using the harmonic probing method, as proposed by Billings and Tsang (1989).
2.2. NARX with finite nonlinear memory

NARX model may be understood as a branch of the neural network technique that is well suited for the identification of a nonlinear dynamic system with a given input and output data. The term, ‘AR’, stands for auto-regressive, which means that the current output of a system is given as a function of the previous system output history. ‘X’ stands for exogenous, meaning that in addition to the past system output, the past external excitation acting on the system also influences the current system output. Eq. (5) presents the mathematical expression of NARX model of a system, which is defined as an unknown function, \( F \), the target to identify using the system identification technique.

\[
\hat{y}_t = F(y, u) = F(y_{t-1}, y_{t-2}, \ldots, y_{t-K}, u_t, u_{t-1}, u_{t-2}, \ldots, u_{t-L}),
\]

where \( \hat{y}_t \) is the predicted system response at time \( t \) and two vectors, \( y \) and \( u \), are the time delayed, true system output and external excitation vectors during some previous time span. Therefore, \( u_{t-L} = u(t-i) - n_0 \Delta t, y_{t-K} = y(t-i) - n_0 \Delta t \), where \( n_0 \) and \( n_b \) are the delay length of the external excitation and system response, respectively. The TDNN is recovered when the auto-regressive part is omitted from the NARX. Note that the NARX model generally outperforms TDNN in terms of the predictability (Chance et al., 1998) because the NARX have infinite memory length owing to the recursive relationship between the current system output and the past one. Depending on the phenomenon that is targeted to be analyzed, the memory effect induced by the nonlinearity of a system may be assumed to be either zero or finite, whereas the memory effect of linearity is kept to be either finite or infinite. This assumption was found to be true and quite effective in predicting the nonlinear dynamic response of a ship (Adegeest, 1997) and jacket structure (Najafian, 2007). The current study proposes an efficient NARX model with finite nonlinear memory to identify the system of a slender marine structure exposed to a Morison type wave loading. This dramatically simplifies the system identification process because the recursive relationship of system output is limited to the linear part, leaving the nonlinear response as a function of external excitation only. Fig. 1 illustrates schematically the NARX model with finite nonlinear memory. The nonlinear block takes the external excitation only, whereas the linear block takes both the excitation and past response. Owing to the recursive relation of the response, the memory becomes infinite for linear part, whereas it becomes finite for the nonlinear part.

The NARX model with finite nonlinear memory, as illustrated in Fig. 1, can be represented mathematically as Eq. (6).

\[
y(t) = (y - r_n)P_yL + (u - r_n)P_uL + \sum_{k=1}^{N} a_k f[(y - r_n)Qb_k + c_k],
\]

where the vector \( L \) and \( b_k \) are both the linear and nonlinear network coefficients, and the vectors, \( r_y \) and \( r_u \), are the mean values of \( y \) and \( u \), respectively. The matrices \( P_y \), \( P_u \) and \( Q \) are linear and nonlinear subspace, which are used to accelerate the convergence of the nonlinear least square method. \( a_k \) is the output network coefficient, \( c_k \) is the static bias vector and \( N \) is the number of neurons. Eq. (6) indicates that the recursive relationship of the system output is not included in the nonlinear activation function, \( f[\cdot] \), but is relevant only to the linear part. The choice of this activation function is rather free because one will end up with different weight factors if a different activation function is chosen, but producing the same output. In this sense, the form of the functional relationship between the input and output is not considered to be unique, but it targets the true unique relationship approximately. In this study, the sigmoidal function given in Eq. (7) is used as a nonlinear activation function.

\[
f(x) = \frac{1}{1 + e^{-x}}
\]

Once the model structure is determined, as given in Eq. (6), the next step is to determine the network parameters through minimization of the sum square errors between the model predicted output and true output. Essentially, this is a well-known nonlinear least square problem, so the aim is to minimize the error function given in Eq. (8).

\[
E(\overline{\sigma}) = \sum_{n=1}^{M} (y_n - \hat{y}_n)^2,
\]

where \( y_n \) is the network predicted system output and \( \hat{y}_n \) is the true output. \( \overline{\sigma} \) is the vector that contains the network parameters as its components.
The least square problem shown in Eq. (8) is nonlinear in terms of the network parameter owing to the sigmoidal nonlinear activation function; hence, the modified version of Gauss–Newton method, called Levenberg–Marquardt method, was applied to find the minimum point over the given error function. The Gauss–Newton method is an approximate version of the original Newton’s method to alleviate the computational burden to evaluate the Hessian at every iteration by relating the Hessian matrix to the Jacobian matrix of the given function to be minimized. The Levenberg–Marquardt method modified the original Gauss–Newton method by adding additional term on top of the Jacobians to accelerate the convergence. According to the Levenberg–Marquardt method, the position is updated following the rule given as

\[
\begin{align*}
  x_{k+1} &= x_k - H^{-1}(x_k)g(x_k), \\
  H(x_k) &= 2J^T(x_k)J(x_k),
\end{align*}
\]

where \( g \) is the gradient vector of the error function, \( H \) is the Hessian matrix and \( J \) is the Jacobian matrix of the error function.

### 2.3. Extraction of transfer functions

Once the system is fully identified and represented by a mathematical expression, one can extract the transfer functions of the system by probing the system using simple harmonic excitations. This is called the harmonic probing method that was originally proposed by Bedrosian and Rice (1971) for the continuous time series. The methodology was later extended to the discrete time series by Billings and Tsang (1989). The basic idea of the method is to probe the system with simple harmonic inputs and match it with the same harmonic components of the expected system outputs. Chance et al. (1998) applied this method to the general NARX model and derived the transfer functions of a system up to the third order.

Before probing the system with simple harmonics, the system equation given in Eq. (6) needs to be simplified further, so that the frequency matching of all terms are possible. To do that, the given sigmoidal function should be expanded to polynomials using the Taylor series; hence

\[
\sum_{k=1}^{N} a_k f((u - r_{0})Q_{b_{k}} + c_{k}) = \sum_{k=1}^{N} a_k (uQ_{b_{k}} + \gamma_{k}) = \sum_{k=1}^{N} \frac{\gamma_{k}^{(n)}}{n!} \left(\frac{\gamma_{k}}{c_{k}}\right)^n,
\]

where \( c_{k} - r_{0}Q_{b_{k}} \) was replaced by \( \gamma_{k} \) for brevity. Combining both linear and nonlinear parts after truncating the series at the second order gives

\[
\begin{align*}
  y_i &= (y - r_{y})P_{y}L + (u - r_{u})P_{a}L + \sum_{k=1}^{N} a_k f(\gamma_{k}) \\
  &\quad + \sum_{k=1}^{N} a_k f'(\gamma_{k})(uQ_{b_{k}}) + \frac{1}{2} \sum_{k=1}^{N} a_k f''(\gamma_{k})(uQ_{b_{k}})^2.
\end{align*}
\]

To extract the LTF, the system is probed with a harmonic of unit magnitude with its circular frequency of \( \omega \) as follows:

\[
x(t) = e^{j\omega t}.
\]

Because the system in consideration may be arbitrarily nonlinear under this monochromatic excitation, the output of the system can be expressed as Eq. (13).

\[
y(t) = H_{1}(\omega)e^{j\omega t} + H_{2}(\omega, \omega)e^{2j\omega t} + H_{3}(\omega, \omega, \omega)e^{3j\omega t} + \ldots
\]

To simplify the derivation process, a delay operator needs to be introduced so that the delayed system output and external excitation can be expressed in simple forms. The delay operators are defined as

\[
\Delta_{o} = [1, \ e^{-j\omega\Delta t}, \ e^{-j2\omega\Delta t}, \ e^{-j3\omega\Delta t} \ldots e^{-jN\omega\Delta t}], \\
\Delta_{w} = [e^{-j\omega\Delta t}, \ e^{-j2\omega\Delta t}, \ e^{-j3\omega\Delta t} \ldots e^{-jN\omega\Delta t}].
\]

Hence, the delayed system output and the monochromatic external excitation can be expressed as

\[
\begin{align*}
  y &= H_{1}(\omega)e^{j\omega t}\Delta_{w}^{1} + H_{2}(\omega, \omega)e^{2j\omega t}\Delta_{w}^{2} + H_{3}(\omega, \omega, \omega)e^{3j\omega t}\Delta_{w}^{3} + \ldots \\
  u &= e^{j\omega t}\Delta_{w}^{0}.
\end{align*}
\]

After substituting Eq. (15) into Eq. (11), the LTF can be derived by matching the harmonics of the same frequency that appear in the both sides of the equation. For the LTF, only the harmonics of the frequency \( \omega \) are relevant; hence

\[
H_{1}(\omega) = \frac{\Delta_{o} P_{a} L}{1 - \Delta_{w}^{1} P_{y} L} + \sum_{k=1}^{N} a_k f(\gamma_{k})\Delta_{o}^{k}Q_{b_{k}}.
\]

For the QTF, the system is probed with two harmonics of difference frequencies, i.e., bichromatic excitation, as shown in Eq. (17).

\[
x(t) = e^{j\omega_{1} t} + e^{j\omega_{2} t}.
\]
Now the expected system output is
\[ y(t) = H_1(\omega_1)e^{i\omega_1 t} + H_1(\omega_2)e^{i\omega_2 t} + 2H_2(\omega_1, \omega_2)e^{i(\omega_1 + \omega_2)t} + \ldots \]  
(18)

The substitution of Eq. (15) into Eq. (11) leads to the QTF by matching the harmonics of \( \omega_1 + \omega_2 \) appearing in the both sides of the equation; hence, the QTF can be expressed as
\[ H_2(\omega_1, \omega_2) = \frac{\sum_{k=1}^{N} a_k f^r(\gamma_k) \left( \Delta_{\omega_1}^0 Q_b_k \right)}{1 - \Delta_{\omega_1 + \omega_2}^1 P_1 L}. \]  
(19)

The cubic transfer function, or even higher order transfer functions may be derived in a similar manner so that the proposed model can cover even a stronger nonlinear problem that the dynamic response of the slender marine structure under a Morison load. Such an example would contain the nonlinear response of a structure influenced by the hydrodynamic splash load near the free surface.

3. Application to slender marine structure

3.1. Application model and time domain analysis

A simple 2-dimensional catenary riser model exposed to the Morison load was chosen to validate the proposed methodology. Two catenary risers connected on top are assumed to stand on the sea floor without a floater attached. The water depth is assumed to be 448 m, which is same as the height of the structure and the horizontal distance from the center to the far end is 224 m, allowing the length of the riser to be 519.4 m. The cross section of the riser was assumed to be a circular hollow pipe made of mild steel, and the outer diameter was 400 mm with a wall thickness of 16 mm. The riser is pin-connected to the sea floor and the top of the riser is fixed vertically (Fig. 2). Even though this model is not a complete representative of the real riser system, the case study with the simplified model may provide a chance to assess the ability of the proposed method to deal with the nonlinear terms of the Morison formula and also to give an insight into the nonlinear dynamic behavior of slender marine structures, including the identification of the influence of the difference frequency components on the nonlinear behavior of the system.

The external load acting on the structure is a wave induced Morison load, which is composed of a transverse drag force, \( F_D \), and inertia forces, \( F_I \), both of them are defined as
\[ F_D = \frac{1}{2} \rho C_D D V_R |V_R|, \]
\[ F_I = \frac{1}{2} \rho C_I D^2 A, \]  
(20)
where \( \rho \) is the density of water and \( D \) is the diameter of the riser. \( V_R \) is the relative transverse velocity of the water particle and structure and \( A \) the water particle acceleration. \( C_D \) and \( C_I \) is the drag coefficient and inertia coefficient, which were set to 0.7 and 1.5, respectively. The water particle velocity and acceleration was determined based on the linear airy wave theory. The steady current load was not considered because the focus was only on the dynamic response of the system under wave load. The time series of the random ocean wave was generated based on the JONSWAP spectrum (DNV, 2000) with a random phase angle under the assumption that the wave elevation follows a Gaussian distribution. Fig. 3 shows 9 wave spectra used for the analysis, where the significant wave height varied 4.5, 8.5 and 12.5 m with its modal period of 8, 10 and
12 s, respectively, with a peakedness factor of 1.0. Direct nonlinear time domain analysis was carried out using the commercial software, ABAQUS Ver.6.10.

The riser experiences large displacement particularly due to the slowly varying difference frequency component coming from the quadratic term included in the drag force term of the Morison formula; hence, the nonlinearities of the considered system are both the quadratic excitation of the wave induced drag and the large displacement of the structure.

For the 9 wave spectra given, the direct nonlinear implicit time domain analysis was performed for 3 h, and the horizontal displacement of the riser top was monitored under the given wave load as datasets for the system identification.

The riser was modeled with 2-node beam finite elements and the number of elements for each riser was 35. To consider the possible large displacement of the riser, the geometric nonlinear effect was taken into consideration. Fig. 4 shows the time history of the horizontal displacement of the riser top under two different wave spectra, one with $H_s=4.5$ m and $T_p=8$ s, and the other one with $H_s=12.5$ m and $T_p=12$ s.

Table 1 lists the maximum, mean and root mean square of the horizontal displacement of the riser top for all 9 wave spectra. As expected, the maximum and mean together with the root mean square of the response tend to increase with increasing significant wave height. Note that the non-zero mean response of the structure is induced by the difference frequency component of the drag force, which is proportional to the square of the wave height. The response also tends to increase as the modal wave period becomes larger because the dynamic effect increases due to the high flexibility of the risers.

![Fig. 3. Wave spectra.](image)

![Fig. 4. Horizontal displacement of the riser top for the two wave spectra. (a) $H_s=4.5$ m, $T_p=8$ s. (b) $H_s=12.5$ m, $T_p=12$ s.](image)

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Maximum/mean/root mean square of the horizontal displacement.</th>
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<tbody>
<tr>
<td>$H_s$</td>
<td>Max/mean/rms</td>
</tr>
<tr>
<td>4.5 m</td>
<td>2.007/0.089/0.318</td>
</tr>
<tr>
<td>8.5 m</td>
<td>4.643/0.389/0.664</td>
</tr>
<tr>
<td>12.5 m</td>
<td>9.041/0.938/1.142</td>
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3.2. System identification

For system identification purposes, the wave elevation at a reference location and the horizontal tip displacement of the riser under two selected wave spectra were used as the training dataset. The selected wave spectra are those of the 10 s modal period and 4.5/12.5 m significant wave heights. Two wave spectra of different significant wave heights were used for identification because of the significantly different nonlinear characteristics of the two cases. The aim was that the system responses of the remaining 7 cases would be predicted after the system was identified using the selected two datasets. The time series of both wave elevation and horizontal displacement during the first 50 min out of 3 h simulation time was used as the training data because a longer data length did not improve the prediction considerably. The memory length of both the linear and nonlinear part was set to 20, and the number of neurons was set to 40. The time increment used for the implicit dynamic analysis was 0.04 s, and the data was sampled at intervals of 5, so that the memory length was 4 s in terms of time. Note that the memory length of 4 s eventually turned out to be sufficient to model the nonlinear part of the system. This is because the structural response induced by the nonlinear wave excitation, particularly the slowly varying drag force, is close to the quasi-static phenomenon, i.e., a dynamic problem with a short memory effect. As stated before, the linear part still has infinite memory length owing to the recursive relationship of the system output.

Fig. 5 shows the open loop prediction results, where the prediction of the system response was made based on the known past system output and external excitation without feedback of the system output. From an application point of view, this open loop prediction is of no use because the system output is not priori known. On the other hand, it indicates the performance of the NARX model by comparing the model predicted output with the true system output. The correspondence between the prediction and target is almost perfect, so that the model is considered to be sufficiently accurate.

Figs. 6 and 7 show the linear and quadratic transfer functions, which were derived using the relationship given in Eqs. (16) and (19), respectively. The modulus of the LTF continuously increases with decreasing frequency, which indicates that the natural frequency of the structure is quite small due to the flexibility of the structure with a long and slender nature. Regarding the QTF, the modulus of the sum frequency is negligibly small in terms of its magnitude throughout the entire sum frequency plane, whereas that of the difference frequency maintains considerable magnitude, especially around the diagonal of the plane. A small area of a relatively high modulus on the sum frequency plane is present near the origin, or \( \omega_1 + \omega_2 \approx 0 \); however, it is expected that this will not contribute to the response significantly because the wave energy is almost absent in this area. On the other hand, the modulus of difference frequency is concentrated along the diagonal, or \( \omega_1 - \omega_2 \approx 0 \), so that the response is expected to be influenced heavily by the difference frequency components. Moreover, the area of concentration overlaps the frequency range of high wave energy. The mean response of the system is also expected from the frequency combinations along the diagonal of the difference frequency plane, i.e. \( \omega_1 - \omega_2 = 0 \).

Fig. 8 compares the time series of the prediction and simulation results under two wave spectra used as the training data during the identification. The plot interval is between 5000 and 5700 s, which is the time period outside of the training data spanning the first 50 min, i.e., 0–3000 s. The time series of the prediction was generated using the Volterra series given in Eq. (4), where both the transfer functions and wave excitations are involved. Fig. 8 shows that the slowly varying quadratic response is present for both cases and it strongly affects the accuracy of the prediction, particularly when the wave height is...
large. In the case of $H_s = 4.5$ m, the nonlinearity involved in the system response is rather small, as expected. Once the second order solution is added to the first order solution, the prediction compares fairly well with the simulation results. Fig. 9 compares the probability of exceeding the peaks, which was calculated from both the simulation and prediction for the entire simulation time span of 3 h. The time series prediction using the Volterra series corresponds well with simulation results for both cases. The probability plot shows that the peaks of the case with $H_s = 12.5$ m, $T_p = 10$ s are slightly underestimated, especially when the magnitudes are large, but the difference is not considerable.

3.3. Response predictions

As the system under consideration is fully identified using the NARX and harmonic probing method, the nonlinear response of the structure under arbitrary random wave excitation can be predicted using the obtained frequency response.
functions and external wave excitations. Of the 9 sea states considered in this study, the time series of the response for the remaining 7 sea states were predicted using the quadratic Volterra series and the results were compared with the time domain simulation results.

Fig. 10 shows the time history of horizontal displacement of riser top for the remaining seven sea states. The simulation results and the solution of each order, along with the summation of the two solutions were compared. The time window between 5000 and 5700 s was selected to provide the details of the time series. Regardless of the sea states considered in the analysis, the first order solution failed to reproduce the slowly varying response of the structure induced by the fluid drag force, which is also the case for the previous two sea states. On the other hand, when the second order components were added to the first order solution, the predicted time history of the response corresponds fairly well with the simulation result. As expected, the contribution of the second order solution increases with increasing wave height. The second order solution shown in Fig. 10 clearly shows that the nonlinearity is mainly that of a slowly varying response, which is excited by the difference frequency component of the drag force. In addition, the rapidly varying response excited by the sum frequency component of the drag force is damped out quickly so that the effect of it is not influential. Overall, the geometric nonlinear effect of the structural response is limited to the second order level under these particular conditions in an approximate sense.

Table 2 summarizes the correlation coefficients of the predicted and simulated signals for all sea states considered in the study, excluding those two sea states used for training. To consider the potential influence of the training data length, two different case studies were carried out, one with a training data length of 50 min, and the other with 90 min. The correlation
A coefficient was defined to be the inner product of the two signals of entire length normalized by the product of the norms of the two signals, so that a coefficient of 1 means a perfect match between the two signals. In all cases considered in this study, the correlation coefficient was higher than 0.93, meaning that the predicted signal is very close to the simulated one.

The influence of the training data length between 50 min and 90 min was not considerable, but some improvements were observed.

Fig. 10. Time series predictions for the other 6 wave spectra. (a) $H_s = 4.5$ m, $T_p = 8$ s. (b) $H_s = 8.5$ m, $T_p = 8$ s. (c) $H_s = 12.5$ m, $T_p = 8$ s. (d) $H_s = 4.5$ m, $T_p = 12$ s. (e) $H_s = 8.5$ m, $T_p = 12$ s. (f) $H_s = 12.5$ m, $T_p = 12$ s.

The correlation coefficient was defined to be the inner product of the two signals of entire length normalized by the product of the norms of the two signals, so that a coefficient of 1 means a perfect match between the two signals. In all cases considered in this study, the correlation coefficient was higher than 0.93, meaning that the predicted signal is very close to the simulated one. The influence of the training data length between 50 min and 90 min was not considerable, but some improvements were observed.
Fig. 11 shows the probability of exceedance of the peaks. Again, the peaks were selected out of the entire simulation time span of 3 h to check the performance of the prediction. The first order solution fell far short of the simulation results, but once the second order solution was added, the gap between the simulation and prediction almost disappeared. An interesting aspect of Fig. 10 is the fact that the predicted minima are not as accurate as the peaks. In terms of the absolute magnitude, the peaks are sometimes over-predicted, whereas the minima are under-predicted. This may be due to the less accurate prediction of the second order solution, which is in charge of the local mean level of the response. Fig. 11 shows a clear tendency that the predicted peaks are larger than the simulation results when the peak magnitudes are of a moderate level. Owing to the limited predictability of the second order Volterra series, care should be taken when making a long-term prediction of the range of the structural response, which is critical when the fatigue life is of primary concern. For an accurate prediction of the range of the structural response, the third order solution needs to be considered. Kim et al. (2014) discussed the effects of the third order solution, where the third order Volterra model was applied to predict the nonlinear structural response of the large container carrier voyaging through the irregular seaway.

Fig. 12 compares the power spectral density of both the prediction and simulation results for the two particular sea states. The general tendency regarding the power spectral density was that compared to the simulation results, the predicted results underestimated the response of the wave frequency region, whereas they overestimated the response of the low frequency region.

4. Conclusions

This paper proposed an efficient system identification method of the finite memory quadratic Volterra model for the time series prediction of the nonlinear structural response of a slender marine structure exposed to a Morison load. Several important conclusions are made.

- NARX with a finite nonlinear memory effect was proposed and applied to a simple riser structure exposed to the Morison load of a random ocean wave. The nonlinear structural response under an arbitrary random ocean wave could be predicted accurately once the system was fully identified using the NARX model of finite nonlinear memory.
- The mathematical relationship between the parameters of the NARX and quadratic Volterra series was derived relying on the harmonic probing method, so that both linear and quadratic transfer functions can be calculated directly using the identified parameters of NARX. The relationship between the NARX parameters and the transfer functions of the system was simpler than that of NARX with infinite memory owing to the absence of a recursive term in the nonlinear part.
- The structural system composed of two catenary risers was identified using NARX with finite memory using the selected datasets of two short term sea states. Both LTF and QTF were obtained using the derived relationship between the transfer functions and NARX parameters. The system responses of the two selected cases were reproduced accurately using the quadratic Volterra series.
- The quadratic transfer function showed that the difference frequency components close to zero are the main contributors to the nonlinear response of the structure under consideration. The time series representation also showed that the slowly varying components were the major portion of the second order solution. The expected response induced by the sum frequency component was not significant because the modulus of the QTF was concentrated in the frequency range where the wave energy is quite low.
- The system responses of the other 7 sea states were predicted using the quadratic Volterra series. The correspondence between the simulated and predicted system response was excellent for all 7 sea states tested. The predictability of the model was not degraded even in case of the sea states of different modal periods than that used for identification.
- The proposed methodology can be used to screen the design load cases, or configuration optimization of offshore structure exposed to a random ocean wave load. The performance of the methodology will not deteriorate even in cases where the model includes the floater, because the motion of the floater is usually calculated up to the second order component as far as the current design practice is concerned.

<table>
<thead>
<tr>
<th>Significant wave height</th>
<th>Training time</th>
<th>$T_p = 8$ s</th>
<th>$T_p = 10$ s</th>
<th>$T_p = 12$ s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_s = 4.5$ m</td>
<td>50 min</td>
<td>0.9316</td>
<td>–</td>
<td>0.9386</td>
</tr>
<tr>
<td></td>
<td>90 min</td>
<td>0.9365</td>
<td>–</td>
<td>0.9468</td>
</tr>
<tr>
<td>$H_s = 8.5$ m</td>
<td>50 min</td>
<td>0.9590</td>
<td>0.9522</td>
<td>0.9393</td>
</tr>
<tr>
<td></td>
<td>90 min</td>
<td>0.9602</td>
<td>0.9542</td>
<td>0.9472</td>
</tr>
<tr>
<td>$H_s = 12.5$ m</td>
<td>50 min</td>
<td>0.9601</td>
<td>–</td>
<td>0.9428</td>
</tr>
<tr>
<td></td>
<td>90 min</td>
<td>0.9601</td>
<td>–</td>
<td>0.9483</td>
</tr>
</tbody>
</table>
When the structural deformation increases beyond the second order level, a third order solution may need to be considered. The extension of the proposed model to the third order is straightforward because of the simplicity of the relationship between the frequency response functions and NARX parameters.

Fig. 11. Probability of exceedance of peaks for the other 6 wave spectra (red—simulation, black—1st+2nd order, blue—1st order). (a) $H_s = 4.5 \text{ m, } T_p = 8 \text{ s}$. (b) $H_s = 8.5 \text{ m, } T_p = 8 \text{ s}$. (c) $H_s = 12.5 \text{ m, } T_p = 8 \text{ s}$. (d) $H_s = 4.5 \text{ m, } T_p = 12 \text{ s}$. (e) $H_s = 8.5 \text{ m, } T_p = 12 \text{ s}$. (f) $H_s = 12.5 \text{ m, } T_p = 12 \text{ s}$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
Fig. 12. Comparison on the power spectral densities (red solid – simulation, black dot – prediction). (a) \(H_s = 8.5\) m, \(T_p = 8\) s. (b) \(H_s = 12.5\) m, \(T_p = 8\) s. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

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