Lateral pressure effect on average compressive strength of stiffened panels for in-service vessels

Joonmo Choung, Jun-Bum Park & Chang Yong Song

To cite this article: Joonmo Choung, Jun-Bum Park & Chang Yong Song (2014) Lateral pressure effect on average compressive strength of stiffened panels for in-service vessels, Ships and Offshore Structures, 9:1, 110-118, DOI: 10.1080/17445302.2012.736360

To link to this article: http://dx.doi.org/10.1080/17445302.2012.736360

Published online: 21 Nov 2012.

Submit your article to this journal

Article views: 76

View related articles

View Crossmark data

Citing articles: 2 View citing articles
Lateral pressure effect on average compressive strength of stiffened panels for in-service vessels

Joonmo Choung*, Jun-Bum Park and Chang Yong Song

*Department of Naval Architecture and Ocean Engineering, INHA University, Incheon, Korea; Korea Technology Centre, Lloyd’s Register Asia, Busan, Korea; Department of Ocean Engineering, Mokpo National University, Jeonnam, Korea

(Received 24 August 2011; final version received 16 August 2012)

This paper presents the estimation of the average compressive strengths of three types of stiffened panels under axial compression and lateral pressure based on simplified formulae in the CSR for tankers (common structural rules for double hull oil tankers) and nonlinear finite element analyses (FEAs). Basic scenarios are determined based on the slenderness ratios of the stiffened panels used for in-service ships. Secondary scenarios are subdivided by external pressures that are applied to finite element model by increasing 1 bar, assuming 30 m water height. The total number of FEAs for flat bar (FB)-, angle bar (AB)- and tee bar (TB)-stiffened panels is 189. FEA results show that the existence of pressure can cause a significant reduction of ultimate strength, while CSR formulae do not take into account the effect of lateral pressure. The lateral pressure is more detrimental to the ultimate strength of stiffened panels with a higher column slenderness ratio than those with a smaller column slenderness ratio. A new concept of the relative average compressive strain energy, instead of the ultimate strength, is introduced in order to rationally compare the average compressive strength through a complete compressive straining history. The differences in the ultimate strengths between CSR formulae and FEA results are relatively small for FB- and AB-stiffened panels, but larger discrepancies of relative average compressive strain energies are obtained.

Keywords: stiffened panel; average compressive strength; ultimate strength; slenderness ratio; relative average compressive strain energy; lateral pressure

1. Introduction

The primary reasons to recognise the compressive strength of a stiffened panel arise from two aspects: local and global strengths. The ultimate strength of the stiffened panel under the combination of uniaxial compressive load, biaxial load, lateral pressure, shear load, etc. is important in the view of local structures, while the average compressive strength of each stiffened panel element constituting a hull section affects the global hull girder bending capacity. Stiffened panels in deck or bottom structure are believed to be more important than those in different area, because they are located at higher position and, as a result, contribute to hull girder longitudinal strength.

A number of studies have dealt with compressive strengths in the fields of naval architecture and ocean engineering. Several simplified formulae have been proposed to facilitate estimation of the ultimate strength of stiffened panels (Lin 1985; Lee 1989; Cho et al. 1998; Paik and Kim 2002; Paik and Thayamballi 2003). In the context of the average compressive strength, relatively fewer studies have been carried out (Gordo et al. 1996; Cho et al. 1998; Paik and Kim 2002; Paik and Thayamballi 2003). Updated studies on weld-induced residual stress and initial deflection are reported in Khan and Zhang (2011) and Pretheesh et al. (2010), respectively. Recently, due to the compulsory installation of permanent means of access (PMA), the ultimate and compressive strengths were reviewed for some perforated panels (Kim et al. 2009). Simplified formulae corresponding to several collapse modes were suggested for the average compressive strength of the stiffened panel (IACS 2010a, 2010b).

The goal-based standard (GBS) issued by the International Maritime Organization (IMO) specifies that the longitudinal residual ultimate strength of a global hull girder with major damage and the ultimate strength of a local stiffened panel with minor damage should be verified in the design and construction stage of the vessel. The latter pertains to the structural redundancy of the stiffened panel and should satisfy the relevant functional requirements. The reduced ultimate strength of the damaged stiffened panel should fulfil structural redundancy. Meanwhile, the former corresponds to the longitudinal residual strength.

According to the CSR procedure, estimation of the average compressive strength of the stiffened panel is critical for the hull girder ultimate strength. In this paper, the CSR formulae used in the incremental-iterative approach or progressive collapse method (PCM) are verified by a finite element analysis (FEA) under a combination of axial compression and lateral pressure.

*Corresponding author. Email: cysong@mokpo.ac.kr

© 2012 Taylor & Francis
There are several components affecting shortening behaviours of stiffened panels: loading combination, boundary conditions and slenderness ratios of the panel. Since loading combination and boundary conditions have less scope of change, parametric study with variation of slenderness ratios and lateral pressures is carried out using non-linear FEAs. It should be noted that the main framework of this paper is based on two references of Nam et al. (2010) and Choung et al. (2011).

2. CSR simplified formulae

Two CSRs (IACS 2010a, 2010b) recommend using the incremental-iterative approach to estimate the ultimate bending moment of the hull structure and provide simplified formulae with several collapse modes as follows:

\[ \sigma_c = \Phi \sigma_{c1} \frac{A_s + A_{pe}}{A_s + A_p} \]  
\[ \sigma_c = \Phi \frac{A_s \sigma_{c2} + A_p \sigma_{ep}}{A_s + A_p} \]  
\[ \sigma_c = \Phi \sigma_0 \frac{b_x t_p + b_w t_w + b_f t_f}{b t_p + b_w t_w + b_f t_f} \]  
\[ \sigma_c = \Phi \frac{A_p \sigma_{ep} + A_s \sigma_{c4}}{A_p + A_s} \]  
\[ \sigma_c = MIN \left( \Phi \sigma_0 \left[ \frac{b}{a} \left( \frac{2.25}{\beta_e} - \frac{1.25}{\beta_e^3} \right) \right] + 0.1 \left( 1 - \frac{b}{a} \right) \left( 1 + \frac{1}{\beta_e^2} \right)^2, \Phi \sigma_0 \right) \]  
\[ \Phi = \begin{cases} \varepsilon & \text{for } 0.0 \leq \varepsilon \leq 1.0 \\ 1.0 & \text{for } \varepsilon > 1.0 \end{cases} \]

\( \sigma_c \): average compressive strength of stiffened panel  
\( \sigma_{c1} \): elastic buckling strength of stiffened panel with column collapse mode based on effective width of plate  
\( \sigma_{c2} \): elastic buckling strength of stiffened panel with lateral torsion collapse mode based on entire width of plate  
\( \sigma_{c4} \): elastic buckling strength of stiffener web with local collapse mode based on entire web height  
\( \sigma_{ep} \): elastic buckling strength of all edge simply supported plate based on effective width of plate  
\( \sigma_0 \): initial yield strength  
\( \Phi \): coefficient representing a strain magnitude  
\( \varepsilon \): relative average compressive strain that equals to average compressive strain divided by yield strain = \( \varepsilon_c / \varepsilon_0 \)  
\( \varepsilon_c \): average compressive strain  
\( \varepsilon_0 \): initial yield strain corresponding to initial yield strength  
\( A_s \): sectional area of stiffener  
\( A_{pe} \): effective sectional area of plate  
\( A_p \): sectional area of plate  
\( a \): length of stiffened panel, which usually equals the frame space  
\( b \): entire width of plate  
\( b_c \): effective width of plate  
\( t_p \): thickness of plate  
\( b_w \): entire height of stiffener web  
\( b_{wc} \): effective height of stiffener web  
\( t_w \): thickness of stiffener web  
\( b_f \): width of stiffener flange  
\( t_f \): thickness of stiffener flange  
\( \beta_e \): slenderness ratio of plate based on entire width of plate member, which includes \( \varepsilon \).

Equation (1) corresponds to the column collapse mode of the stiffened panel, which is induced by axial compressive loads in a simply supported column on both ends. Equation (2) accommodates the lateral torsional collapse mode induced by moments in a simply supported column on both ends. Equation (4) represents the tripping collapse mode of a flat bar (FB) stiffener, while the tripping collapse mode of angle bar (AB) and tee bar (TB) stiffeners can be derived by Equation (3). Equation (5) implies the plate collapse mode without any stiffener; hence, it should be applied to a separate plate member. Equation (6) is a coefficient representing the magnitude of the relative average compressive strain. A pertinent collapse mode is dominantly decided by selecting a simplified formula that produces the minimum ultimate strength.

3. Distributions of slenderness ratios

The buckling and ultimate strengths of a stiffened panel can be characterised by three design variables, called slenderness ratios: plate slenderness ratio \( \beta \), stiffener slenderness ratio \( \lambda_t \), and stiffened panel slenderness ratio \( \lambda_c \). The formulae representing these three slenderness ratios are given in Equations (7)–(9). In the context of the slenderness formulae, both elastic column buckling and elastic lateral torsion buckling strengths are presented in Equation (10) and Equation (11), respectively. \( \sigma_c \) and \( \sigma_{ec} \) are similar to each other, but they are based on different plate widths. On the other hand, \( \sigma_{c2} \) and \( \sigma_{c4} \) are identical formulae that are based on the entire width of the plate.

\[ \beta = \frac{b}{t_p} \sqrt{\frac{\sigma_0}{E}} \]  
\[ \lambda_c = \frac{\sigma_0}{\sigma_{ec}} \]  
\[ \lambda_t = \frac{\sigma_0}{\sigma_{ct}} \]  
\[ \sigma_{ec} = \frac{\pi^2 EI}{Aa^2} \]
\[
\sigma_{et} = \frac{1}{I_p} \left( GJ + \pi^2 EI_w \frac{a^2}{a^2} \right) \tag{11}
\]

\[\beta: \text{slenderness ratio of plate} \]
\[\lambda_c: \text{slenderness ratio of stiffened panel} \]
\[\lambda_t: \text{slenderness ratio of stiffener} \]
\[\sigma_{et}: \text{elastic buckling strength with column collapse mode} \]
\[\sigma_{et}: \text{elastic buckling strength with lateral torsional collapse mode} \]
\[\eta: \text{area second moment of inertia about major principal axis} \]
\[\sigma_{et}: \text{elastic buckling strength with column collapse mode based on entire width of plate} \]
\[\lambda_c: \text{slenderness ratio of stiffener web} \]
\[\lambda_t: \text{slenderness ratio of stiffened panel} \]
\[\sigma_{et}: \text{elastic buckling strength with lateral torsional collapse mode based on entire width of plate} \]
\[I: \text{polar moment of inertia about intersection point between plate and stiffener web} \]
\[J: \text{polar moment of inertia about shear centre of stiffened panel (torsion constant)} \]
\[I_w: \text{warping moment of inertia about intersection point between plate and stiffener web} \]
\[E: \text{elastic modulus of material} \]

\(\beta\): slenderness ratio of plate
\(\lambda_c\): slenderness ratio of stiffened panel
\(\lambda_t\): slenderness ratio of stiffener
\(\sigma_{et}\): elastic buckling strength with column collapse mode based on entire width of plate
\(\sigma_{et}\): elastic buckling strength with lateral torsional collapse mode based on entire width of plate
\(I\): area second moment of inertia about major principal axis
\(\eta\): sectional area of stiffened panel
\(I_p\): polar moment of inertia about intersection point between plate and stiffener web
\(J\): polar moment of inertia about shear centre of stiffened panel (torsion constant)
\(I_w\): warping moment of inertia about intersection point between plate and stiffener web
\(E\): elastic modulus of material

Nam et al. (2010) and Choung et al. (2011) presented the probabilistic values such as average (AVG), lower limit (LL) and upper limit (UL) of the slenderness ratios, as given in Table 1. The UL and LL imply the average plus and minus two times the standard deviation, respectively. A stiffened panel with a large value of \(\beta\) will show plate collapse mode, while a large value of \(\lambda_c\) will lead to tripping collapse mode. Column collapse mode is usually observed in the case of a large value of \(\lambda_t\).

### 4. Finite element analyses

The FEAs are carried out by using Abaqus/Standard (Simulia 2008) and the stiffened panel is composed of four node shell elements with the reduced integration scheme (S4R), which implies there is only one integration point at the centre of the element. Sine waves of Equations (12) and (13) are applied for the initial deflections of the plate and stiffener due to fabrication and transportation, respectively. Smith et al. (1988) proposed Equation (14) with three levels of plate deflection. The average level is applied to the FE model in this paper. In Equations (12) and (13), 0.15% of the frame space is the magnitude of the initial out-of-plane deflection in the plate-stiffener web intersection line and the stiffener web, as delineated in Equation (15) (DNV 2009).

\[
\delta_p(x, y) = a_p \sin \frac{m \pi x}{a} \sin \frac{\pi y}{b} + a_s \sin \frac{\pi x}{a} \tag{12}
\]

\[
\delta_w(x) = a_w \sin \frac{\pi x}{a} \tag{13}
\]

For severe level \(\beta\) is applied to the FE model in this paper. In Equations (12) and (13), 0.15% of the frame space is the magnitude of the initial out-of-plane deflection in the plate-stiffener web intersection line and the stiffener web, as delineated in Equation (15) (DNV 2009).

\[
\delta_p(x, y) = a_p \sin \frac{m \pi x}{a} \sin \frac{\pi y}{b} + a_s \sin \frac{\pi x}{a} \tag{12}
\]

\[
\delta_w(x) = a_w \sin \frac{\pi x}{a} \tag{13}
\]

\[x: \text{coordinate symbol representing longitudinal direction of stiffened panel} \]
\[y: \text{coordinate symbol representing transverse direction of stiffened panel} \]
\[\delta_p: \text{out-of-plane deflection of plate} \]
\[\delta_w: \text{out-of-plane deflection of web} \]
\[a_p: \text{magnitude of initial deflection of plate} \]
\[a_w: \text{magnitude of lateral deflection of stiffener} \]
\[m: \text{number of half waves in longitudinal direction} \]

Considering residual force equilibrium of a welded stiffened panel, Equations (16)–(18) present half or entire widths of the tensile residual stress zones of the plate, stiffener web and stiffener flange, respectively. The compressive residual stress is presented as Equation (19) (Smith et al. 1988), where the average level is applied to the FE model in this paper. Namely, the compressive residual stress is assumed to be 15% of the initial yield strength.

\[
\eta_p = \frac{\sigma_r b}{\sigma_0 + \sigma_r} \tag{16}
\]

\[
\eta_w = \begin{cases} 
\frac{\sigma_r b_w}{\sigma_0 + \sigma_r} & \text{for } FB \\
\frac{\sigma_r b_w}{\sigma_0 + \sigma_r} & \text{for } AB \text{ and } TB 
\end{cases} \tag{17}
\]

\[
\eta_f = \begin{cases} 
\frac{\sigma_r b_f}{\sigma_0 + \sigma_r} & \text{for } AB \\
\frac{\sigma_r b_f}{\sigma_0 + \sigma_r} & \text{for } TB 
\end{cases} \tag{18}
\]

\[
\sigma_r = \begin{cases} 
-0.05 \sigma_0 & \text{for slight level} \\
-0.15 \sigma_0 & \text{for average level} \\
-0.30 \sigma_0 & \text{for severe level}
\end{cases} \tag{19}
\]
Figure 1. FE model and nodes for boundary conditions. (This figure is available in colour online.)

\[ \eta_p: \text{half width of tensile residual stress zone of plate} \]
\[ \eta_w: \text{width or half width of tensile residual stress zone of web} \]
\[ \eta_f: \text{width or half width of tensile residual stress zone of flange} \]
\[ \sigma_r: \text{compressive residual stress} \]

In this paper, the FEA model consists of four transverse bays and seven longitudinal bays to minimise the effects of boundary conditions, as shown in Figure 1. The foremost and aftermost transverse bays (first T. Bay and fourth T. Bay, respectively) are dummies to compensate the effects of the boundary conditions, while the second and third transverse bays represent stiffener-induced and plate-induced collapse modes, respectively. The FE model includes initial deflections determined from Equations (12)–(15). Elements with narrow width representing tensile residual stress zone are modelled based on Equations (16)–(19). Lengths of most elements are kept to be 100 mm with unit aspect ratio, except for the tensile residual stress zone.

With length \( a \) and width \( b \) fixed at 3200 mm and 800 mm, respectively, the other dimensions are adjusted to obtain the target slenderness ratios, as delineated in Table 1. The elastic modulus and initial yield strength are 206 GPa and 315 MPa and strain hardening is not taken into account.

The boundary conditions considering the continuity with adjacent stiffened panels are described in Table 2, where subscripts \( x, y \) and \( z \) are fully compliant with the direction symbols in Figure 1. Symbols \( T \) and \( R \) in Table 2 mean translational and rotational constraints, respectively. The transverse frames are not included in the FEA model but are substituted by the boundary conditions. All degrees of freedom (DOF) except for the loading direction are constrained by using rigid elements in both loading sides to prevent local collapse in way of both loading edges due to the compressive loads.
5. Evaluation of average compressive strengths

The ultimate strengths calculated from FEAs and CSR simplified formulae are presented in Figure 3. The stiffened panels F5-5 and A5-5 are collapsed by ±3 bar pressures, hence no results are plotted in Figure 3. The minimum ultimate strengths among the different collapse modes from the CSR formulae are suggested as solid symbols in Figure 3. The ultimate strengths from the CSR formulae are not changed according to the pressure variation, because the pressure effect is not included in the CSR simplified formulae. However, the ultimate strengths from the FEAs are seriously aggravated by the pressure. Since the slenderness ratios of FB- and AB-stiffened panels vary over a much wider range than those of TB-stiffened panels, the ultimate strengths of the corresponding stiffened panels worsen more with the pressure. For the stiffened panels with small $\lambda_c$, the pressure exerts little effect on the ultimate strength. For example, ultimate strengths of F1-1, F3-1 and F5-1 or A1-1, A3-1 and A5-1 do not change rapidly. On the other hand, as $\lambda_c$ becomes slender, the pressure results in a sudden decrease of the ultimate strength. Consequently, it is recommended that stiffened panels with small $\lambda_c$ be used for the bottom shell and inner bottom shell, which are exposed to large sea water and internal cargo pressures, respectively.

The CSR formulae tend to overestimate the ultimate strengths in comparison to the FEA, and the discrepancies between the CSR formulae and FEA are exacerbated by

Table 2. Applied boundary conditions.

<table>
<thead>
<tr>
<th>Boundary conditions</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unloaded plate edges</td>
<td>$T_z = R_z = R_t = 0$</td>
</tr>
<tr>
<td>Loaded edges</td>
<td>$T_z = T_r = R_z = R_t = 0$</td>
</tr>
<tr>
<td>Frame–plate intersection lines</td>
<td>$T_z = R_z = 0$</td>
</tr>
<tr>
<td>Frame–web intersection lines</td>
<td>$T_z = R_t = 0$</td>
</tr>
<tr>
<td>Frame–flange intersection lines</td>
<td>$T_t = R_t = 0$</td>
</tr>
</tbody>
</table>

Lateral pressure and shortening load are given as two load steps, as presented in Table 3 and Figure 2, where second load step includes pressure and axial compression. To exclude the interaction of compressive load and lateral pressure in the load step 1, the lateral pressure is applied in the load step 1 and the compressive load is added to the load step 2. Prescribed displacements of ±100 mm are applied to the aftermost and foremost edges, respectively.

The basic criteria to generate the analysis scenarios in Table 4 are $\beta$ and $\lambda_c$; $\lambda_t$ is excluded because inclusion of $\lambda_t$ frequently produces imaginary values or unrealistic values of geometric dimensions. The stiffened panels composing the side shell and bulkheads experience lateral pressure due to external sea water or internal liquid freight. Neglecting the dynamic pressure effect, the lateral pressure is about maximum ±3 bar, supposing that the maximum water height is about 30 m. In this paper, ±3 bar pressures are sequentially applied by 1 bar, where positive pressure implies the plate to stiffener direction. As a result, FEAs are carried out for the seven pressure load cases described in Table 4. Total number of nonlinear FEAs is 189. On average, each analysis takes calculation time of less than 10 minutes.

Table 3. Applied loading conditions.

<table>
<thead>
<tr>
<th>Load step 1</th>
<th>$±3$ bar on plate face</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load step 2</td>
<td>$T_z = 100$ mm on the loaded edges</td>
</tr>
</tbody>
</table>

Table 4. Analysis scenarios for FEA.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\lambda_c$</th>
<th>Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1-1</td>
<td>LL</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>F1-3</td>
<td>LL</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>F1-5</td>
<td>LL</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>F3-1</td>
<td>AVG</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>F3-3</td>
<td>AVG</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>F3-5</td>
<td>AVG</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>F5-1</td>
<td>UL</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>F5-3</td>
<td>UL</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>F5-5</td>
<td>UL</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>A1-1</td>
<td>LL</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>A1-3</td>
<td>LL</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>A1-5</td>
<td>LL</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>A3-1</td>
<td>AVG</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>A3-3</td>
<td>AVG</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>A3-5</td>
<td>AVG</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>A5-1</td>
<td>UL</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>A5-3</td>
<td>UL</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>T1-1</td>
<td>LL</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>T1-3</td>
<td>LL</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>T1-5</td>
<td>LL</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>T3-1</td>
<td>AVG</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>T3-3</td>
<td>AVG</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>T3-5</td>
<td>AVG</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>T5-1</td>
<td>UL</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>T5-3</td>
<td>UL</td>
<td>$±3$ bar by 1 bar</td>
</tr>
<tr>
<td>T5-5</td>
<td>UL</td>
<td>$±3$ bar by 1 bar</td>
</tr>
</tbody>
</table>
the pressure. It is inferred that the CSR formulae neglect initial imperfections such as initial deflection and residual stress or are developed based on the slight levels of initial imperfections. As seen in Figure 3, the distribution of ultimate strengths shows a symmetric pattern under positive and negative pressures; however, the ultimate strengths under negative pressure are slightly smaller than those under positive pressure.

The CSR recommends ceasing iteration of the multi-step approach in order to obtain the hull girder bending moment-curvature curve once the hogging or sagging curvature of the hull girder theoretically reaches the final curvature, which corresponds to three times the yield moment. This implies that the average compressive capacity up to the final curvature is of importance. It is noted that, in practice, the ultimate strength among Equations (1)–(5) has served as a criterion to select a rational collapse mode, but the accuracy of CSR simplified formulae should be verified throughout the entire strain regime. Both the ultimate and post-ultimate strengths of the stiffened panel substantially affect the hull girder bending capacity. For this reason, a dimensionless average strain energy concept, given by Equation (20), may be a better measure for the mode selection than the ultimate strength. Since the relative average strain energy is obtained from integration of the average compressive strength curve to the termination strain, it can represent the comprehensive strength history including the ultimate strength and post-ultimate strength. In addition, the relative average strain energy may be used for verification of the CSR formulae:

$E_r = \int_{0.0}^{3.0} \left( \sigma_c / \sigma_0 \right) d \left( \varepsilon_c / \varepsilon_0 \right)$  \hspace{1cm} (20)

$E_r$: dimensionless relative average strain energy

Under the assumption that the FEA results are reliable, the preferable collapse mode of the CSR formulae can be determined from the mode selection by comparing the relative average strain energies from the FEA and CSR formulae. If there are no FEA results, the collapse mode can be practically decided based on the minimum relative average energy.

The comparative results of the ultimate strength and the relative average compressive strain energy for FB-, AB- and TB-stiffened panels are presented in Tables 5–7, wherein the error refers to the differences between the FEA and CSR results. For direct comparison of the FEA and CSR results, FEA results in Tables 5–7 do not include the lateral pressure.

From Table 5, the average error of the ultimate strength for the FB-stiffened panel is relatively small (about 4.8%), and thus it is possible to state that the CSR results agree relatively well with the FEA results. Neither the standard deviation (SD) nor the coefficient of variation (COV) of error is sufficiently large. However, the average error of the relative average compressive strain energy reaches about 27.6% and the SD and COV of error become much larger than those of the ultimate strength. It is concluded that the CSR simplified formulae may overestimate the post-ultimate compressive strength of the FB-stiffened panel.
Table 5. Comparison of ultimate compressive strength and relative average compressive strain energy from CSR and FEA for FB-stiffened panels.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_u/\sigma_0$ from FEA</th>
<th>$\sigma_u/\sigma_0$ from CSR</th>
<th>Error of $\sigma_u/\sigma_0$</th>
<th>$E_r$ from FEA</th>
<th>$E_r$ from CSR</th>
<th>Error of $E_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1-1</td>
<td>0.8890</td>
<td>0.8455</td>
<td>0.0489</td>
<td>2.4939</td>
<td>2.0900</td>
<td>0.1620</td>
</tr>
<tr>
<td>F1-3</td>
<td>0.8977</td>
<td>0.9188</td>
<td>0.0236</td>
<td>1.6307</td>
<td>2.3900</td>
<td>0.4656</td>
</tr>
<tr>
<td>F1-5</td>
<td>0.8580</td>
<td>0.8082</td>
<td>0.0580</td>
<td>1.3169</td>
<td>2.4700</td>
<td>0.8756</td>
</tr>
<tr>
<td>F3-1</td>
<td>0.8011</td>
<td>0.7734</td>
<td>0.0346</td>
<td>1.9049</td>
<td>1.6900</td>
<td>0.1128</td>
</tr>
<tr>
<td>F3-3</td>
<td>0.8472</td>
<td>0.8960</td>
<td>0.0576</td>
<td>1.5421</td>
<td>2.0900</td>
<td>0.3553</td>
</tr>
<tr>
<td>F3-5</td>
<td>0.8112</td>
<td>0.7860</td>
<td>0.0312</td>
<td>1.2039</td>
<td>2.1800</td>
<td>0.8108</td>
</tr>
<tr>
<td>F5-1</td>
<td>0.6894</td>
<td>0.6394</td>
<td>0.0726</td>
<td>1.7190</td>
<td>1.3300</td>
<td>0.2263</td>
</tr>
<tr>
<td>F5-3</td>
<td>0.7307</td>
<td>0.7828</td>
<td>0.0713</td>
<td>1.5558</td>
<td>1.7600</td>
<td>0.1313</td>
</tr>
<tr>
<td>F5-5</td>
<td>0.7140</td>
<td>0.6905</td>
<td>0.0329</td>
<td>1.1307</td>
<td>1.8200</td>
<td>0.6096</td>
</tr>
<tr>
<td>Mean</td>
<td>0.8043</td>
<td>0.7934</td>
<td>0.0478</td>
<td>1.6109</td>
<td>1.9800</td>
<td>0.4169</td>
</tr>
<tr>
<td>STDEV</td>
<td>0.0726</td>
<td>0.0844</td>
<td>0.0171</td>
<td>0.3900</td>
<td>0.3421</td>
<td>0.2757</td>
</tr>
<tr>
<td>COV</td>
<td>0.0903</td>
<td>0.1063</td>
<td>0.3571</td>
<td>0.1728</td>
<td>0.6619</td>
<td></td>
</tr>
</tbody>
</table>

Table 6. Comparison of ultimate compressive strength and relative average compressive strain energy from CSR and FEA for AB-stiffened panels.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_u/\sigma_0$ from FEA</th>
<th>$\sigma_u/\sigma_0$ from CSR</th>
<th>Error of $\sigma_u/\sigma_0$</th>
<th>$E_r$ from FEA</th>
<th>$E_r$ from CSR</th>
<th>Error of $E_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1-1</td>
<td>0.5823</td>
<td>0.9387</td>
<td>0.6121</td>
<td>1.3164</td>
<td>2.1900</td>
<td>0.6636</td>
</tr>
<tr>
<td>A1-3</td>
<td>0.9071</td>
<td>0.9589</td>
<td>0.0571</td>
<td>2.2193</td>
<td>2.2400</td>
<td>0.0993</td>
</tr>
<tr>
<td>A1-5</td>
<td>0.8733</td>
<td>0.8915</td>
<td>0.0208</td>
<td>1.4481</td>
<td>2.2100</td>
<td>0.5261</td>
</tr>
<tr>
<td>A3-1</td>
<td>0.7185</td>
<td>0.8605</td>
<td>0.1976</td>
<td>1.6449</td>
<td>1.9700</td>
<td>0.1976</td>
</tr>
<tr>
<td>A3-3</td>
<td>0.7931</td>
<td>0.8151</td>
<td>0.0277</td>
<td>1.8919</td>
<td>1.8300</td>
<td>0.0327</td>
</tr>
<tr>
<td>A3-5</td>
<td>0.7715</td>
<td>0.8038</td>
<td>0.0418</td>
<td>1.4102</td>
<td>1.8200</td>
<td>0.2906</td>
</tr>
<tr>
<td>A5-1</td>
<td>0.6753</td>
<td>0.8033</td>
<td>0.1895</td>
<td>1.5338</td>
<td>1.8400</td>
<td>0.1997</td>
</tr>
<tr>
<td>A5-3</td>
<td>0.7198</td>
<td>0.6714</td>
<td>0.0672</td>
<td>1.6918</td>
<td>1.4600</td>
<td>0.1370</td>
</tr>
<tr>
<td>A5-5</td>
<td>0.6962</td>
<td>0.7162</td>
<td>0.0287</td>
<td>1.4142</td>
<td>1.5300</td>
<td>0.0819</td>
</tr>
<tr>
<td>Mean</td>
<td>0.7486</td>
<td>0.8288</td>
<td>0.1380</td>
<td>1.6190</td>
<td>1.8989</td>
<td>0.2376</td>
</tr>
<tr>
<td>STDEV</td>
<td>0.0947</td>
<td>0.0900</td>
<td>0.1794</td>
<td>0.2696</td>
<td>0.2679</td>
<td>0.2105</td>
</tr>
<tr>
<td>COV</td>
<td>0.1265</td>
<td>0.1085</td>
<td>1.2994</td>
<td>0.1665</td>
<td>0.1411</td>
<td>0.8861</td>
</tr>
</tbody>
</table>

Table 6 shows that the average error of the relative average strain energy of the AB-stiffened panel from the CSR simplified formulae is greatly increased compared with that of the ultimate strength. In the case of the TB-stiffened panel, the average errors from both CSR simplified formulæ and FEA results are almost coincident with respect to the ultimate strength and the relative average compressive strain energy. As previously mentioned, the reason for the small error is that the distribution of the slenderness ratio of the TB-stiffened panel is very narrow.

Table 7. Comparison of ultimate compressive strength and relative average compressive strain energy from CSR and FEA for TB-stiffened panels.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_u/\sigma_0$ from FEA</th>
<th>$\sigma_u/\sigma_0$ from CSR</th>
<th>Error of $\sigma_u/\sigma_0$</th>
<th>$E_r$ from FEA</th>
<th>$E_r$ from CSR</th>
<th>Error of $E_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1-1</td>
<td>0.8557</td>
<td>0.9124</td>
<td>0.0663</td>
<td>1.9898</td>
<td>2.0900</td>
<td>0.0503</td>
</tr>
<tr>
<td>T1-3</td>
<td>0.8691</td>
<td>0.9135</td>
<td>0.0511</td>
<td>2.0400</td>
<td>2.1200</td>
<td>0.0392</td>
</tr>
<tr>
<td>T1-5</td>
<td>0.8648</td>
<td>0.9182</td>
<td>0.0617</td>
<td>2.0411</td>
<td>2.1200</td>
<td>0.0387</td>
</tr>
<tr>
<td>T3-1</td>
<td>0.8129</td>
<td>0.8566</td>
<td>0.0538</td>
<td>1.8208</td>
<td>1.9900</td>
<td>0.0929</td>
</tr>
<tr>
<td>T3-3</td>
<td>0.8285</td>
<td>0.8549</td>
<td>0.0319</td>
<td>1.9497</td>
<td>1.9700</td>
<td>0.0104</td>
</tr>
<tr>
<td>T3-5</td>
<td>0.8235</td>
<td>0.8613</td>
<td>0.0459</td>
<td>1.9673</td>
<td>1.9700</td>
<td>0.0014</td>
</tr>
<tr>
<td>T5-1</td>
<td>0.7796</td>
<td>0.8008</td>
<td>0.0272</td>
<td>1.7794</td>
<td>1.8700</td>
<td>0.0509</td>
</tr>
<tr>
<td>T5-3</td>
<td>0.7942</td>
<td>0.7957</td>
<td>0.0018</td>
<td>1.8845</td>
<td>1.8300</td>
<td>0.0289</td>
</tr>
<tr>
<td>T5-5</td>
<td>0.7881</td>
<td>0.8049</td>
<td>0.0213</td>
<td>1.9451</td>
<td>1.8300</td>
<td>0.0592</td>
</tr>
<tr>
<td>Mean</td>
<td>0.8240</td>
<td>0.8576</td>
<td>0.0401</td>
<td>1.9353</td>
<td>1.9767</td>
<td>0.0413</td>
</tr>
<tr>
<td>STDEV</td>
<td>0.0316</td>
<td>0.0467</td>
<td>0.0198</td>
<td>0.0859</td>
<td>0.1098</td>
<td>0.0255</td>
</tr>
<tr>
<td>COV</td>
<td>0.0384</td>
<td>0.0545</td>
<td>0.4945</td>
<td>0.0444</td>
<td>0.0556</td>
<td>0.6181</td>
</tr>
</tbody>
</table>
For three different types of stiffeners, maximum values of errors of \( E_r \) are found for cases of F1-5, A1-1 and T3-1. Case of T3-1 provides less than 10% error of \( E_r \), while relatively larger errors of \( E_r \) are witnessed for cases of F1-5 and A1-1. For this reason, shortening histories for cases of F1-5 and A1-1 are depicted in Figure 4 where deformed shapes are presented together.

First investigating F1-5, error of ultimate strengths from FEA and CSR mode 1 formula is approximately 5.8%, but the reason why this case shows large error of \( E_r \) is well described in Figure 4a. The post-ultimate strength by CSR mode 1 formula is far overestimated compared to one by FEA. Figure 4a primarily shows column collapse mode, but it appears that tripping collapse shape is also combined to primary mode.

Overall histories of shortening for A1-1 panel show remarkable difference between FEA and CSR mode 3. Deformed shape expressed in Figure 4b reveals that three members of plate, web and flange simultaneously start to buckle and reach final collapse. This leads premature collapse of the stiffened panel.

Considering pressure effect from external sea water or liquid cargo, the difference of prediction between FEA and CSR formulae becomes larger. Therefore, it is thought that the accuracy of the CSR formulae needs to be improved or a new formulation for the average compressive strength curve is required.

6. Concluding remarks

The ultimate strength of the stiffened panel is a measure of the local collapse strength under the combination of compressive load, biaxial load, lateral pressure, shear load, etc., while the average compressive strength of the stiffened panel is required to predict the global hull girder ultimate strength by using an incremental-iterative approach. This paper, focusing on the prediction of the global hull girder strength by PCM, estimates the average compressive strength of a stiffened panel subject to both lateral pressure and compressive load by using nonlinear FEAs and CSR simplified formulae.

This paper refers to the probabilistic distributions of the slenderness ratios of a plate, stiffener and stiffened panels from Nam et al. (2010). The upper, average and lower limits of the plate and stiffened panel slenderness ratio distributions produce nine basic load cases for nonlinear FEAs.
Including incremental pressure cases based on a 30 m water height to each basic load case, 189 nonlinear FEAs are carried out for FB-, AB- and TB-stiffened panels.

It is verified from the FEAs that the ultimate strength is easily degenerated by application of pressure, while CSR formulae do not take into account any pressure effect. Since the slenderness ratios of FB- and AB-stiffened panels are widely distributed, the degradation of the ultimate strengths of these panels appears to be accelerated compared to TB-stiffened panels. In addition, the existence of higher pressure can worsen the ultimate strengths of the stiffened panels with small \( \lambda_c \). Accordingly, it is required that the stiffened panels are selected, carefully, to achieve a sufficient value of \( \lambda_c \); especially, the stiffened panels in way of bottom shell and inner double bottom because they are subjected to the large pressure.

In order to estimate the hull girder ultimate strength by using the incremental-iterative approach, the compressive strength history of the stiffened panel is more reasonable than the ultimate strength. Hence, this paper proposes that the relative average compressive strain energy is considered for the evaluation of the compressive strength capacity of a stiffened panel.

It is inferred that the difference in the ultimate strengths from FEAs and CSR formulae comes from the degree of initial imperfections. The CSR formulae may be based on slight initial imperfections. The difference in the relative average compressive strain energies from FEAs and CSR formulae is larger than that of the ultimate strengths. The CSR formulae may not accurately simulate rapid load shedding in the post-ultimate strength region.

The ultimate hull girder bending capacity is strongly dependent on accurate estimation of the average compressive strength of the stiffened panel. Hence, it is necessary to study not only the effects of the average compressive strength on the hull girder ultimate strength but also the modified formulation of the average compressive strength accommodating the effects of lateral pressure.

Acknowledgements
This work is the outcome of a Manpower Development Program for Marine Energy by the Ministry of Land, Transport and Maritime Affairs (MLTM). This work was also supported by the New & Renewable Energy of the Korea Institute of Energy Technology Evaluation and Planning (KETEP) grant funded by the Korea government, Ministry of Knowledge Economy. The authors are grateful to INHA University for funding research grant. This research was also supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2012R1A1A1002897).

References