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Failure Behavior of the Piezoelectric Materials under Purely Electrical Loading

by

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Failure Behavior of the Piezoelectric Materials under Purely Electrical Loading
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ABSTRACT

In this thesis, failure behavior of piezoelectric materials under purely electrical loading is investigated. Under purely electrical loading, piezoelectric material commonly shows two kinds of failure modes: fracture by crack growth and dielectric breakdown by formation of tubular channel or conducting path. Both unpoled and poled piezoelectric materials are considered in this study. The problems of the tubular channel growth in unpoled and poled PZT807 piezoelectric ceramics with cylindrical bar shape under electric fields, which have not been solved yet, are experimentally and numerically analyzed. Also, the interface failure problem of PZT C-201/C-3 piezoelectric bimaterial is experimentally and theoretically investigated. Finally, kinking of a conducting path between two dissimilar anisotropic dielectric materials, which can be considered as the unpoled piezoelectric materials, is analyzed using linear transformation method.

The dielectric breakdown of an unpoled piezoelectric ceramic, PZT807, with a conductive channel, is investigated. Cylindrical bar specimens with a conductive channel are used for breakdown tests of the unpoled piezoelectric ceramic under purely electrical loads. Narrow tubular channels emanating from the head of the initial channel are observed in the specimens after breakdown occurs. The radius of the tubular channel that is created at the surface of the initial channel head is insensitive to various types of channel formation. The problem of a fine tubular channel that emanates from the initial channel head is numerically solved to evaluate the three-dimensional $J$ integral, which is directly related to the energy that is available at breakdown, at the initiation of a new channel in the specimen. The critical $J$ integral at the onset of breakdown is obtained.

Tubular channel growth in a piezoelectric material with a conductive channel is investigated. Breakdown tests are performed on PZT807 samples with cylindrical bar shapes under purely electrical loading. It is experimentally observed that dielectric
breakdown occurs via the formation of tubular channels, and the new tubular channel propagates in a straight direction through the specimen. The three-dimensional $J$ integral for a tubular channel is used as a criterion for dielectric breakdown failure. The $J$ integral at the onset of breakdown is calculated numerically through finite element analysis. The critical $J$ integrals at the onset of breakdown are obtained.

The failure behavior under purely electric loading of piezoelectric ceramics with a conductive crack is investigated. The study lies on the experimental observation made by 박재언 (2003). According to his observation, two failure phenomena, fracture and dielectric breakdown, are occurred. And the dielectric breakdown occurs via the formation of tubular channels and that the radii of the tubular channels are almost independent of the poling direction and the direction of electric loading. The critical $J$ integrals at the onset of both fracture and breakdown are calculated numerically via finite element analysis. The effects of both the direction of the electric field and the poling direction on both fracture and breakdown resistance are discussed. The possibilities of fracture and dielectric breakdown are also discussed.

Fracture behavior of a piezoelectric bimaterial with an interfacial electrode subjected to purely electric loading is investigated. Plate shape specimens with interfacial electrodes are used for electrical fracture tests. Cracks emanating from the tip of interfacial electrodes are experimentally observed. The mutual integral, which has the conservation property, is applied to obtain electric field intensity factors due to purely electrical loading for interfacial electrodes between dissimilar piezoelectric materials. The critical electric field intensity factors at the onset of crack growth are obtained.

Kinking of a conducting path between two dissimilar anisotropic dielectric materials under purely electric loading is analyzed. Dielectric breakdown commonly occurs in the dielectric materials by formation of conducting path under a critical electric loading. The problem is formulated using a linear transformation method. Based on the formulation, the complete electric fields in the anisotropic dielectric materials are easily
obtained from the solutions of the isotropic dielectric materials. The electric field intensity factors and energy release rates of kinked conducting path are obtained for various inclination angles of material principal axes. Numerical analysis is conducted to examine the validity of the analytic solution, which is obtained based on the linear transform method, using finite element method. It is seen that the numerical solution is very agree with analytic solution.

*Keywords:* Dielectric breakdown, Piezoelectric material, Tubular channel, $J$ integral, Finite element analysis, Interfacial electrode, Conducting path, Linear transformation method.
본 논문에서는 전기하중을 받는 압전재료의 파괴모동에 대하여 연구하였다. 전기하중하의 압전재료는 흔히 균열성장에 의한 파괴와 관형채널 혹은 전도체적의 형성에 의한 절연파괴 등 두 가지 파괴모드를 보이고 있다. 본 논문연구에서는 비분극과 분극 압전재료가 고려되었다. 아직 풀린 바 없는 원기둥모양을 갖는 비분극과 분극 PZT807 압전세라믹에서의 관형진도체널성 장문제를 실험과 수치해석을 병행하여 해석하였다. 또한 PZT C-201/C-3 이중 압전재료의 계면파괴문제에 대하여 실험과 이론을 병행하여 연구하였다. 마지막으로 산형변환방법을 이용하여 이방성 유전재료에서 발생하는 몇 가지 2 차원 절연파괴문제를 해석하였다. 이방성 유전재료는 압전효과를 제거함으로써 비분극 압전재료로 간주할 수 있다.

진도체널을 갖는 PZT807 비분극 압전세라믹의 절연파괴문제를 연구하였다. 진도체널을 갖는 원기둥모양의 시료를 사용하여 전기하중하에서 비분극 압전재료의 절연파괴 실험을 진행하였다. 절연파괴 후 초기진도체널선단으로부터 좁은 관형체널이 생성하였음을 관찰할 수 있었다. 실험을 통하여 초기 진도체널선단에서 생성되는 관형체널의 반지름은 관형체널의 형성유형과 무관함을 알 수 있었다. 절연파괴가 일어나며 할 때의 관형체널의 성장문제를 수치적으로 해석함으로써 관형체널의 단위길이당 방출되는 에너지를 구할 수 있는 3 차원 J 적분을 평가하였다. 절연파괴가 일어나며 할 때의 임계 J 적분을 구하였다.

진도체널을 갖는 압전재료에서의 관형체널의 성장에 대하여 살펴보았다. 전기하중 하에서 원기둥 모양을 갖는 PZT807 시료에 대하여 절연파괴 실험을 수행하였다. 실험을 통하여 절연파괴는 관형체널의 형성에 의하여 발생하며 생성된 관형체널은 직선경로에 따라 시료를 관통하는 것을 관찰할 수 있었다. 관형체널에 대한 3 차원 J 적분을 절연파괴 예측기준으로 사용하였다. 유한요소해석을 통하여 절연파괴가 일어나며 할 때의 J 적분을 수치적으로 계산하였다. 절연파괴가 일어나며 할 때의 임계 J 적분을 구하였다.
전기하중 하에서 전도균열을 갖는 압전세라믹의 파괴행동에 대하여 연구 하였다. 본 연구는 박재연이 수행한 실험결과에 바탕을 두었다. 그의 관찰에 따르면 실험과정에 균열파손과 절연파괴 등 두 가지 파손모드가 나타났다고 한다. 또한 절연파괴는 관형전도체널의 형성에 의하여 발생하며 관형전도체널의 반지름은 전기장의 방향과 분극방향에 거의 의존하지 않는 것으로 나타났다고 한다. 유한요소해석을 통하여 파괴가 일어나려 할 때에 절연파괴가 일어나려 할 때의 임계 J 적분을 각각 수치적으로 계산하였다. 인가된 전기장의 방향과 분극방향이 균열파손저항과 절연파괴저항에 미치는 영향을 논의하였다. 균열파손과 절연파괴의 발생가능성에 대해서도 논의하였다.

전기하중 하에서 계면전극을 갖는 이종압전재료의 파괴행동에 대하여 연구하였다. 계면전극을 갖는 판 모양의 시료를 사용하여 전기파괴실험을 수행하였다. 계면전극 선단으로부터 균열이 진전하는 것을 실험적으로 관찰하였다. 보존특성을 갖는 상호적분을 적용하여 전기하중 하에서의 이종압전재료의 계면전극에 대한 전기장확대계수를 구하였다. 균열이 성장하려 할 때의 임계 전기장확대계수를 구하였다.

전기하중하의 두 가지 서로 다른 이방성 유전체료사이에서의 전도궤적의 맺힘 문제에 대하여 연구하였다. 전도궤적은 임계 전기하중 하에서 전도궤적의 형성에 의하여 발생된다. 본 연구에서는 선형변환방법을 적용하여 전도궤적파괴문제를 공정화하였다. 문제의 공정화를 바탕으로 등방성유전체료의 해로부터 이방성 유전체료에서의 완전한 전기장을 쉽게 구할 수 있다. 구부러진 전도궤적에 대한 전기장확대계수와 에너지해방율을 다양한 재료주축 경사각에 대하여 구하였다. 유한요소법을 적용한 수치해석을 통하여 선형변환방법을 이용하여 구한 해석적인 해의 유효성을 검토하였다. 검토결과 수치 해는 해석적인 해와 완벽하게 일치하였다.

핵심어: 절연파괴, 압전재료, 관형체널, J 적분, 유한요소해석, 계면전극, 전도궤적, 선형변환방법.
CHAPTER 1

INTRODUCTION

1.1 Motivation and Literature Survey

Being smart materials, piezoelectric ceramics have received much attention for several years due to their extensive application in electromechanical devices such as sensors, actuators, acoustic emission transducers, and hydrophones (Newnham and Ruschau, 1991; Uchino, 1993; Trolier-McKinstry and Newnham, 1993). Representative examples of widely used piezoelectric ceramics are barium titanate (BaTiO3), lead zirconate titanate (PZT), and lead lanthanum zirconate titanate (PLZT). Although these materials are marked by high efficiency of performance, they are brittle and susceptible to cracking under external loading. Thus far, considerable attention has been focused on ascertaining fracture mechanisms of piezoelectric materials arising from the propagation of cracks. Experimental studies have been conducted for cracks with electrically conducting or insulating surfaces in piezoelectric materials. Through a conventional fracture test, Park and Sun (1995) investigated the effects of electric fields on the growth of cracks in a PZT-4 ceramic that is subject to both mechanical and electrical loading. They found that a positive electric field along the direction of poling reduces the apparent fracture toughness, while a negative electric field against the direction of poling enhances the apparent fracture toughness. Lynch et al. (1995) performed an indentation test on PLZT ceramics. They experimentally demonstrated that the PLZT ceramic is susceptible to stable crack growth under a cyclic electric field with an amplitude that is either near or above the coercive field. Heyer et al. (1998) performed a four-point bending test on a poled PZT-PIC 151 specimen under combined electrical and mechanical loadings. They found that a positive electric field increases the apparent fracture toughness of a conducting crack that is parallel to the poling
direction, while a negative field tends to reduce toughness. Zhang et al. (2004) experimentally and theoretically studied the failure behavior of depoled and poled piezoelectric ceramics with electrically conductive cracks under electrical and mechanical loadings. Via an experimental study, they verified the theoretical failure criteria that they had derived in their prior investigations from a CFZ (charge-free zone) model that they had proposed. Theoretical studies of fracture in ferroelectric ceramics with a crack have been carried out by many researchers. Pak (1990) obtained the energy release rate for a mode III fracture problem of linear piezoelectric materials using path-independent integral. Suo (1992) developed a fracture mechanics theory to evaluate static toughness and fatigue crack growth for ferroelectric ceramics under small-scale hysteresis. Beom and Atluri (1996, 2002) obtained the closed form solutions of the stress and electric displacement intensity factors for interfacial insulating crack and stress and electric field intensity factors for interfacial conducting crack between two dissimilar anisotropic piezoelectric materials using complex function theory.

Piezoelectric ceramics exhibit another significant mode of failure, namely, dielectric breakdown. Dielectric breakdown refers to the rapid ionization of molecules and the crystalline structure. The breakdown occurs when the electric field that is applied to the piezoelectric material exceeds a critical value called the dielectric strength. The high electric field can induce the dielectric material to partially ionize and begin conducting. The dielectric strength is sensitive to defects such as micro-cracks, internal electrodes and interior pores. The electric field is intensified in the vicinity of defects, as a result of which it can easily reach the limit of resistance of the material to breakdown. The measured dielectric strength of a natural solid with defects is certainly lower than that of a perfect solid without any defects. Dielectric breakdown is a factor that limits the performance of electronic components, and is an issue in the electronic industry. Therefore, research has focused on the development of an appropriate criterion for predicting dielectric breakdown. Using classical fracture mechanics theory, Zeller and Schneider (1984) investigated the problem of aging in solid dielectrics. They suggested
that a partial discharge channel is able to propagate when the release of electrostatic energy that results from channel growth exceeds the energy that is involved in channel formation. Garboczi (1988) developed a linear, dielectric-breakdown electrostatics theory based on a Griffith energy-balance calculation, and applied the theory to a conducting crack in a dielectric medium. Suo (1993) extended the Griffith energy-balance theory to conductive tubular channels in dielectric and ferroelectric ceramics. Satoh et al. (1996) studied the effect of defects on the dielectric strength of SiO$_2$ films. Recently, Beom and Kim (2008) applied the three-dimensional $J$ integral for a conductive tubular channel to analyze the breakdown of a dielectric material. They showed that the $J$ integral has the physical meaning of the energy released per unit length of the channel, as a result of channel growth. All the studies mentioned above have significantly contributed to the prediction of the dielectric breakdown of specific piezoelectric materials. Nevertheless, a unified criterion of dielectric breakdown, which is consistent with various experimental observations of breakdown in piezoelectric ceramics, has not been developed as yet.

Interfacial failure between piezoelectric materials also becomes a factor that degenerate the performance of electronic components, and caused researchers to devote a great effort to study the failure mechanisms (Suo et al., 1992; Beom and Atluri, 1996; Gao and Wang, 2000). Until now, most of these studies have focused either on the insulating interface crack, or on permeable interface cracks. Just few works are devoted to case of conducting crack on the interface between dissimilar piezoelectric materials (Beom and Atluri, 2002; Wang et al., 2003), and the experimental study for a conducting interface crack embedded in dissimilar piezoelectric materials has not been carried out yet.

1.2 Thesis Outline

This thesis has been organized into 8 chapters.

In Chapter 1, motivation of present study is introduced. Also, the literature is
viewed briefly.

In Chapter 2, the basic features of piezoelectric ceramics are introduced. Also, the constitutive relations and the governing equations of piezoelectric ceramics are presented. The path-independent J integral in piezoelectric ceramics are represented to investigate electrical failure behavior of piezoelectric ceramics.

In Chapters 3 and 4, dielectric breakdown of cylindrical bar shape piezoelectric materials, which includes both unpoled and poled types, with a conductive channel under purely electrical loading are investigated.

In Chapter 5, electrical failure behavior of plate shape piezoelectric ceramics with a conductive crack under purely electric fields is investigated.

In Chapter 6, failure behavior of a piezoelectric bimaterial with an interfacial electrode subjected to purely electrical loading is investigated.

In Chapter 7, kinking of a conducting path between two dissimilar anisotropic dielectric materials, which can be considered as the unpoled piezoelectric materials, is analyzed using linear transformation method.

In Chapter 8, the contributions of this thesis to the research of failure behavior of the piezoelectric materials are summarized.
CHAPTER 2
PIEZOELECTRIC CERAMICS

2.1 Basic Features

Simply stated, piezoelectric materials produce an electric field in response to applied stress. This is the direct piezoelectric effect. The electric field is proportional to the force, and it is therefore of opposite sign for compression and tension. There is a converse effect. An applied electric field produces a proportional strain, expansion or contraction depending on polarity.

The most important piezoelectric ceramics crystallize in the perovskite structure as shown in Fig. 2.1. This structure may be described as a simple cubic unit cell with a large cation (Pb\(^{2+}\)) on the corners, a smaller cation (Ti\(^{2+}\)) in the body center, and oxygens (O\(^{2-}\)) in the centers of the faces. Piezoelectrics with a perovskite structure have a Curie point. Below the Curie point they have a tetragonal structure and above the Curie point they transform into a cubic structure. In the tetragonal state, each unit cell has an electric dipole, i.e. there is a small charge differential between each end of the unit cell. A mechanical force (compressive force or tension force) can decrease or increase the separation between the cations and anions which produces an internal field or voltage. Similarly, an electric field against or along the direction of polarization can decrease or increase the separation between the cations and anions which produces a mechanical deformation (compression or tension).

2.2 Basic Equations

Consider the plane-strain problem of piezoelectric ceramics. The in-plane displacements and the electric potential are the function of only in-plane Cartesian
coordinates \( x_1 \) and \( x_2 \). For piezoelectric materials, the electric field vector \( E_i \) and the infinitesimal strain tensor \( \gamma_{ij} \) are represented by the gradient of the electric potential \( \phi \) and the displacement vector \( u_i \), respectively.

\[
\gamma_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \quad E_i = -\phi, \tag{2.1}
\]

Here, the indices range from 1 to 2, and the subscript comma (,) denotes the partial derivative with respect to the in-plane Cartesian coordinates. In this paper, the repetition of an index in a term denotes a summation with respect to that index over its range 1 to 2 for a lowercase script. On the boundary of piezoelectric materials, the traction vector \( t_i \) and the surface charge density \( \omega \) are given by

\[
t_i = \sigma_{ij} n_j, \quad \omega = D_j n_j. \tag{2.2}
\]

Here, \( \sigma_{ij} \) and \( D_j \) are the stress tensor and the electric displacement vector, respectively. \( n_j \) is the unit outward normal vector on the boundary surface. If the material is free of space charge and the body force, the stress tensor and the electric displacement vectors satisfy the equilibrium equations, namely,

\[
\sigma_{ij,j} = 0, \quad D_{j,j} = 0. \tag{2.3}
\]

### 2.3 Constitutive Relations

According to the thermodynamic theory, the stress and the electric displacement in a piezoelectric ceramic are given by (Suo, 1992)

\[
\sigma_{ij} = \partial W / \partial \gamma_{ij}, \quad D_i = -\partial W / \partial E_i. \tag{2.4}
\]
Piezoelectric ceramics are approximately linear when the loading amplitude is small compared to the depoling field. The electric enthalpy density in a linear piezoelectric ceramic is expressed by a quadratic form of

$$ W = \frac{1}{2} c_{ijkl} \gamma_{ij} \gamma_{kl} - \frac{1}{2} \varepsilon_{ima} E_i E_m - \varepsilon_{ima} E_i \gamma_{mn}, $$

(2.5)

where $c_{ijkl}$ is the elastic stiffness, $\varepsilon_{ima}$ is the piezoelectric constant and $\varepsilon_{im}$ is the dielectric constant.

The relations that constitute the piezoelectric ceramic are obtained from (2.5) and (2.4) as

$$ \sigma_{ij} = c_{ijkl} \gamma_{kl} - \varepsilon_{kij} E_k, $$

$$ D_i = \varepsilon_{ima} \gamma_{mn} + \varepsilon_{im} E_m. $$

(2.6)

For the case of unpoled piezoelectric ceramics, since the electro-mechanical coupling effects disappeared, the constitutive equation (2.6) can be reduced as

$$ D_i = \varepsilon_{ij} E_j. $$

(2.7)

2.4 Energy Considerations

The $J$ integral for an elastoplastic material has been widely used as a fracture parameter. Rice (1968) interpreted the $J$ integral as the rate at which energy is released as a result of the extension of a crack in a nonlinear elastic body. Many researchers (Pak, 1990; Suo et al. 1992; Beom and Atluri, 2002) have studied the $J$ integral for a piezoelectric material for analyzing problems related to cracks. The $J$ integral for a piezoelectric material with a conducting crack is defined as
\[ J = \int_{\Gamma} (W n_1 - t_j u_{j,1} + n_j D_j E_i) d\Gamma, \quad i,j=1,2, \]  \hspace{1cm} (2.7)

where, \( n_j \) is the unit outward vector that is normal to an arbitrary contour \( \Gamma \) that encloses the tip of the conducting crack as shown in Fig. 2.2.

Recently, Beom and Kim (2008) applied the three-dimensional \( J \) integral for an electroelastic material to analyze dielectric breakdown. They showed that the \( J \) integral for a conductive tubular channel is equivalent to the energy that is released per unit length of growth in the channel. The three-dimensional \( J \) integral for a conductive tubular channel is defined as

\[ J = \int_{S} (W n_1 - t_j u_{j,1} + n_j D_j E_i) dS, \quad i,j=1,2,3. \]  \hspace{1cm} (2.8)

Here, \( n_j \) is the unit outward vector normal to an arbitrary surface \( S \) that encloses the head of the conductive channel.

Energy release rate is physically equal to \( J \) integral for insulating crack and conducting crack, thus,

\[ G = J, \]  \hspace{1cm} (2.9)

where \( G \) is the energy release rate.

**2.5 Complex Variable Solution**

2.5.1 Insulating Crack

Consider a generalized two-dimensional deformation of a linear anisotropic piezoelectric ceramic with an insulating crack. The constitutive equation (2.6) can be rewritten in the following compact form (Barnett and Lothe, 1975)
\[
\Sigma^0_{ij} = C_{ijkl} v^0_{lm} ,
\]

in which

\[
v^0_M = \begin{cases} 
  u_M, & M = 1, 2, 3 \\
  \phi, & M = 4 
\end{cases},
\]

\[
\Sigma^0_{ij} = \begin{cases} 
  \sigma_{ij}, & J = 1, 2, 3 \\
  D_j, & J = 4 
\end{cases},
\]

\[
C_{ijkl} = \begin{cases} 
  c_{ijkl}, & J, M = 1, 2, 3 \\
  e_{ij}, & J = 1, 2, 3; M = 4 \\
  e_{lm}, & J = 4; M = 1, 2, 3 \\
  -e_{in}, & J, M = 4
\end{cases},
\]

where \( u_m \) is the displacement, \( \phi \) is the electric potential, \( \sigma_{ij} \) is the stress, \( D_j \) is the electric displacement, \( c_{ijkl} \) is the elastic constant, \( e_{ij} \) is the piezoelectric constant, \( e_{in} \) is the dielectric permittivity. In this paper, the repetition of an index in a term denotes a summation with respect to that index over its range 1 to 3 for a lowercase script and 1 to 4 for an uppercase script, unless indicated otherwise. Therefore, the equilibrium equation (2.3) can be rewritten in compact form of

\[
C_{ijkl} v^0_{lm} = 0 .
\]

A general solution for the displacement and the electric potential that satisfy (2.12), and the corresponding stress and electric displacement components, can be written in terms of four arbitrary functions as (Barnett and Lothe, 1975)

\[
v^0_j = 2 \text{Re} \left[ \sum_{M=1}^{4} A^0_{M} f_M (z_M) \right] .
\]
\[
\psi_j^0 = -2 \text{Re} \left[ \sum_{M=1}^{4} B_{jm}^0 f_M(z_M) \right],
\]
\[
\Sigma_{1j}^0 = \psi_{j,2}^0 = -2 \text{Re} \left[ \sum_{M=1}^{4} B_{jm}^0 p_M f_M'(z_M) \right],
\]
\[
\Sigma_{2j}^0 = -\psi_{j,1}^0 = -2 \text{Re} \left[ \sum_{M=1}^{4} B_{jm}^0 f_M'(z_M) \right].
\] (2.13)

Here, Re denotes the real part, prime (') implies the derivative with respect to the associated arguments, and \( \psi_j^0 \) is the generalized stress potential. \( f_M(z_M) \) are analytic in their arguments, \( z_M = x_1 + p_M x_2 \), and \( p_M \) are four distinct complex numbers with positive imaginary parts, which can be solved as the roots of a eighth-order polynomial (Barnett and Lothe, 1975)

\[
\|Q + p(R + R^T) + p^2 T\| = 0,
\] (2.16)

where \( \| \cdot \| \) denotes the determinant of a matrix, superscript \( T \) indicates the transpose of a matrix, and the three matrices \( Q, R \) and \( T \) are defined by \( Q_{JM} = C_{1JM1} \), \( R_{JM} = C_{1JM2} \) and \( T_{JM} = C_{2JM2} \).

We define the real matrices \( L^0 \) and \( M^0 \) as

\[
L^0 = -\left[ \text{Im}(A^0 B^{0-1}) \right]^{-1}, \quad M^0 = -\left[ \text{Re}(A^0 B^{0-1}) \right].
\] (2.17)

where Im denotes the imaginary part. The matrices \( A^0 \) and \( B^0 \) are not unique, in the sense that any arbitrary constant can be multiplied to the eigenvectors, while the two real matrices \( L^0 \) and \( M^0 \) are unique.

For convenience, we consider the single variable vector function, \( f(z) \), defined as

\[
f(z) = \{ f_1(z), f_2(z), f_3(z), f_4(z) \}^T,
\]

\[
z = x_1 + p x_2, \quad \text{Im}(p) > 0.
\] (2.18)
This one-complex-variable approach has been originally introduced by Suo (1990). Once the solution of $f(z)$ to a given boundary value is obtained, we can calculate the field quantities by substituting $z_1, z_2, z_3$ and $z_4$ into each component function.

2.5.2 Conducting Crack

Consider a generalized two-dimensional deformation of a linear anisotropic piezoelectric ceramic with a conducting crack. The general solutions can be derived by considering a linear combination of four arbitrary functions as follows (Beom and Atluri, 2002):

$$
\nu_j = 2 \text{Re} \left[ \sum_{M=1}^{4} A_{JM} f_M(z_M) \right],
$$

$$
\psi_j = -2 \text{Re} \left[ \sum_{M=1}^{4} B_{JM} f_M(z_M) \right].
$$

(2.10)

Here,

$$
\nu_j = \begin{cases} 
u_j, & J = 1,2,3, \\ \psi_0, & J = 4, \end{cases}
$$

$$
\psi_j = \begin{cases} \psi_j^0, & J = 1,2,3, \\ \phi, & J = 4, \end{cases}
$$

$$
A_{JM} = \begin{cases} A_{JM}^0, & J = 1,2,3, \\ -B_{4M}^0, & J = 4, \end{cases}
$$

$$
B_{JM} = \begin{cases} B_{JM}^0, & J = 1,2,3, \\ -A_{4M}^0, & J = 4. \end{cases}
$$

(2.11)

The stress and electric fields are derived in terms of the potential $\psi_j$, as follows:
\[ \Sigma_{1j} = \psi_{j,2}, \quad \Sigma_{2j} = -\psi_{j,1}, \]  
\hspace{0.5cm} (2.12) \]

in which

\[
\Sigma_{1j} = \begin{cases} 
\sigma_{1j}, & J = 1, 2, 3, \\
-E_2, & J = 4,
\end{cases} \\
\Sigma_{2j} = \begin{cases} 
\sigma_{2j}, & J = 1, 2, 3, \\
E_1, & J = 4,
\end{cases} 
\hspace{0.5cm} (2.13) \]

where \( \sigma_{ij} \) is the stress and \( E_i \) is the electric field. \( \text{Re} \) denotes the real part, and \( f_M(z_M) \) are analytic in their arguments, \( z_M = x_i + p_M x_2 \), and \( p_M \) are four distinct complex numbers with positive imaginary parts, which can be solved as the roots of an eighth-order polynomial. The matrices \( A \) and \( B \) in (2.10) are determined by eigenvalue problem, and are not unique. We can define the real matrices \( L \) and \( M \), which will appear latter chapter in this thesis, as follows

\[
L = -[\text{Im}(AB^{-1})]^{-1}, \quad M = -[\text{Re}(AB^{-1})],
\hspace{0.5cm} (2.14) \]

where \( \text{Im} \) denotes the imaginary part. The two real matrices \( L \) and \( M \) are unique.

For convenience, we consider the single variable vector function, \( f(z) \), defined as

\[
f(z) = \{f_1(z), f_2(z), f_3(z), f_4(z)\}^T, \\
z = x_1 + px_2, \quad \text{Im}(p) > 0.
\hspace{0.5cm} (2.15) \]

Once the solution of \( f(z) \) is obtained for a given boundary value problem, a replacement of \( z_1, z_2, z_3 \) and \( z_4 \) should be made for each component function, to calculate the field quantities.
CHAPTER 3

DIELECTRIC BREAKDOWN OF AN UNPOLED PIEZOELECTRIC MATERIAL WITH A CONDUCTIVE CHANNEL

3.1 Introduction

The purpose of present chapter is to investigate the dielectric breakdown of an unpoled piezoelectric ceramic, PZT807, with a conductive channel. Cylindrical bar specimens with a conductive channel are used for breakdown tests of the unpoled piezoelectric ceramic under purely electrical loads. Narrow tubular channels emanating from the head of the initial channel are observed in the specimens after breakdown occurs. The radius of the tubular channel that is created at the surface of the initial channel head is insensitive to various types of channel formation. The problem of a fine tubular channel that emanates from the initial channel head is numerically solved to evaluate the three-dimensional $J$ integral, which is directly related to the energy that is available at breakdown, at the initiation of a new channel in the specimen. The critical $J$ integral at the onset of breakdown is obtained.

3.2 Experiment

3.2.1 Specimen Preparation

The unpoled piezoelectric ceramic of lead zirconate titanate (PZT807, Morgan ElectroCeramics, UK) was used in this study. Fig. 3.1 schematically shows the geometry of the specimen and the electrical loading condition. All specimens were cylindrical with a radius, $R$, of 2.5mm and a length, $L$, of 10mm. An initial channel was
drilled in each specimen. The length of the initial channel, $a$, was 5mm or 7mm. The radius of the initial channel, $c_0$, ranged between 0.3mm and 0.7mm in increments of 0.1mm. During the drilling procedure, severe abrasion occurred on the micro-drill as a result of the high stiffness of the ceramic. The abrasion caused the shape of the tip of the initial channel to differ from specimen to specimen; as the radius of the initial channel increased, the shape of the tip was observed to change from that of a flat head to one of a hemispheroidal head. After the drilling, the specimens were cleaned in distilled water. In order to render the initial channel highly conductive, it was filled with silver paint, which is a kind of colloidal solution that rapidly solidifies at room temperature. In order to ensure there was no air bubble within the channel, the initial channel was refilled with silver paint until no apparent shrinkage was found. The two ends of the specimen also were painted with silver paint to create two electrodes. A high voltage supplier with a maximum of 50kV (Glassman High Voltage, Japan) was used to apply high electric fields.

3.2.2 Procedure for Testing Dielectric Breakdown

The tests of dielectric breakdown were carried out in accordance with the ASTM-D3755-97 standard (2004). During the test, the electrode surface that was connected to the initial channel was always connected with the anode of the high voltage supplier. In order to minimize the effect of surface discharges prior to breakdown, all the experiments were performed by immersing the test specimens in silicone oil. The voltage was increased at a uniform rate of 500V/s from zero until breakdown.

3.2.3 Experimental Results

Narrow tubular channels emanating from the tip of the initial channel were observed in the specimens after breakdown occurred. Figs. 3.2 and 3.3 show patterns of typical
channel formation in the unpoled PZT807. The channel was usually initiated at the tip of the initial channel and propagated in either a straight or an oblique direction. In a few cases, two or more channels were observed. The radius of the tubular channel that was created at the tip of the initial channel was insensitive to various types of channel formation, and was usually about 30μm. Fig. 3.4 illustrates images from a scanning electron microscope of the unpoled PZT807 after the occurrence of breakdown. It is seen from Fig. 3.4 that the tubular channel was formed by melting of the specimen.

The voltages at the time of occurrence of dielectric breakdown were recorded by using a high voltage supplier for the tests of dielectric breakdown. The measured breakdown voltages, $V_{BD}$, which were applied at the specimen electrodes, are shown in Fig. 3.5. There is considerable scatter in the data of breakdown voltages for the unpoled piezoelectric material. The scatter may be attributed to differences in shape of the tip of the head of the initial channel among specimens. The length of the initial channel exerts considerable influence on the breakdown voltage.

### 3.3 Numerical Analysis

Electrical analyses are performed with the commercial software, ABAQUS, for calculating the electric fields for the specimens. The 8-node, biquadratic, axisymmetric, piezoelectric, quadrilateral element, CAX8E, is used in the FEM computation. The dielectric constant of the unpoled PZT807, $\varepsilon = 8.719 \times 10^{-9}$ C/Vm, is used in the numerical analysis. Due to the linearity of the problem, the dependence of the solution on the difference in the electric potential between the electrodes is known. In the present study, the electric boundary condition is taken as follows. The electrode connected to the initial channel is maintained at $\phi = 100$ V, while the opposite electrode is maintained at $\phi = 0$ V, as shown in Fig. 3.6. The problem of a fine tubular channel that emanates from the tip of the initial crack is also numerically solved for evaluating the $J$ integral at the onset of breakdown in the specimen. The calculation of
the $J$ integral is carried out in an auxiliary program.

3.3.1 Electric Field Distribution

In order to investigate the effect of the shape of the channel head on the electric field and dielectric breakdown, the initial tubular channel with the shape of either a hemispherical head or a flat head is considered. The mesh configurations used in the FEM computation are shown in Fig. 3.6. The numerical results of electric fields on the head surface of the initial channel are shown in Fig. 3.7. Here, $E_0$ is the nominal electric field defined as $E_0 = -\Delta \phi / L$, where $\Delta \phi$ is the difference in the electric potential between the electrodes. $\theta$ is the polar angle measured from the positive $x_1$-axis in the spherical coordinates $(r, \theta, x_1)$ and $\rho$ is the perpendicular distance from the $x_1$-axis in the cylindrical coordinates $(\rho, \phi, x_1)$, as shown in Fig. 3.6. It is seen from Fig. 3.7 that the electric field is intensified around the head of the channel. The electric field is much larger than the nominal electric field, $E_0$. The concentrated electric field may initiate the tubular channel on the channel tip. The distribution of the electric field on the surface of the channel head strongly depends upon the shape of the channel head: the maximum electric field occurs at the centre of the hemispherical head surface, whereas the maximum electric field occurs at the edge of the surface of the flat head. This implies that the tubular channel propagates in straight and oblique directions for initial channels with spherical and flat heads, respectively.

3.3.2 $J$ Integral

Assume the formation, shown in Fig. 3.8, of a tubular channel with length $b$ and radius $c$ at the head of the initial channel with length $a$ and radius $c_0$. The $J$ integral is widely used in analyzing problems concerning cracks in elasticity (Rice, 1968), piezoelectrics (Pak, 1990; Beom and Atluri, 1996) and electrostriction (Beom, 1999).
Recently, Beom and Kim (2008) applied the three-dimensional $J$ integral for a conductive channel to analyze breakdown in a dielectric material. They showed that the three-dimensional $J$ integral can be interpreted as the energy released per unit length of the channel, as a result of channel growth. Details of the properties of the $J$ integral have been presented by Beom and Kim (2008). In this problem, the three-dimensional $J$ integral (2.8) that is determined in Section 2.4 is used to analyze the dielectric breakdown problem of an unpoled PZT 807 ceramic with a conductive channel. The unpoled piezoelectric material can be microscopically regarded as a dielectric material. No mechanical loading is applied to the external boundary of the material. The three-dimensional $J$ integral in (2.8) is reduced to

$$J = \int_{S_h} (Wn_i + n_jD_jE_i) dS,$$  \hspace{1cm} (3.1)

where $S_h$ is the surface of the head of the channel with length $b$ and radius $c$. It is noted that the evaluation of the $J$ integral for the channel requires only the knowledge of the distribution of the electric field on the surface of the channel head.

The FEM computation is carried out to obtain the electric field on the surface of the channel head. The values used here for the channel are $L=10$ mm, $R=2.5$ mm, $a=5$ mm, $c_0=0.6$ mm and $c=30\mu$m. The normalized $J$ integral, $J/(\pi c^2 E_0^2)$, is plotted as a function of $b/c$ in Fig. 3.9. The plot shows that as the length $b$ approaches zero, the value of the $J$ integral for the channel with length $b$ approaches that of the $J$ integral for $b=0$. The $J$ integral for the channel at the onset of the initiation of a new channel can be obtained from the solution of the initial channel ($b=0$).

### 3.4 Numerical Results and Discussion

The $J$ integral for the initial channel in the unpoled PZT807 specimen with $c=30\mu$m and $b=0^\circ$ is numerically calculated. For the cases when $a=5$ mm and $a=7$ mm, the
normalized $J$ integral, $J/(\pi a^2 E_0^2)$, is plotted as a function of $c_0$ in Fig. 3.10. Here, a spherical shape for the head of the initial channel is assumed, and values of 10mm and 2.5mm for L and R, respectively, have been used in the computations. Fig. 3.10 shows that the length and radius of the initial channel strongly affect the $J$ integral. The normalized $J$ integral decreases rapidly as the radius of the initial channel increases. The normalized $J$ integral for the initial channel when $a$ is 7mm is much higher than that for the case when $a$ is 5mm.

It is experimentally observed that the dielectric breakdown occurs via the formation of a tubular channel. The $J$ integral is directly related to the energy that is available at breakdown. The $J$ integral may be used as a parameter that governs the breakdown process. When the applied $J$ integral reaches the critical value, $J_c$, dielectric breakdown occurs. The critical $J$ integral at the initiation of breakdown is shown in Fig. 3.11. It is observed experimentally that the value of $c$ is insensitive to $a$ and $c_0$, and is usually about 30$\mu$m. In obtaining the critical $J$ integral at the onset of breakdown, $c=30\mu$m has been used. The experimental result shows that the critical $J$ integral increases as $c_0$ decreases and $a$ increases. $J_c$ is less sensitive to the length of the initial channel, whereas the radius of the initial channel exerts a strong influence on $J_c$. As mentioned in section 3, the $J$ integral depends on the distributions of the electric enthalpy density, electric displacement and electric field on the surface of the channel head. We have assumed that the unpoled material behaves in a linear electric manner. The linear dielectric theory predicts the concentration of the electric field on the surface of the channel head. The electric field around the channel head intensifies as $c_0$ decreases and $a$ increases. After the electric field exceeds a critical magnitude, the electric displacement approaches a finite limit. Due to electric saturation, the electric field around the channel head is redistributed. A domain switch may occur in a zone around the head of the channel in the unpoled ceramic. The strains induced by the domain switch around the channel head generate stresses. These nonlinearities may require modification of the solution of the electric fields in order to use the $J$ integral as a breakdown criterion that
can be associated with significant nonlinearities.

3.5 Summary

The dielectric breakdown of an unpoled piezoelectric ceramic PZT807 with a conductive channel is investigated. Cylindrical bar specimens with a conductive channel are used for tests of the breakdown of the unpoled piezoelectric ceramic under purely electrical loads. Narrow tubular channels emanating from the tip of the initial channel are observed in the specimens after breakdown occurs. The channel is usually initiated at the head of the initial channel and propagates in either a straight or an oblique direction. The radius of the tubular channel that is created at the head of the initial channel is insensitive to various types of channel formation. In order to investigate the effect of the shape of the channel head on the electric field and dielectric breakdown, the initial tubular channel with the shape of either a hemispherical head or a flat head is considered. It is shown that the direction of the channel growth depends upon the shape of the head of the initial channel. The problem of a fine tubular channel emanating from the head of the initial channel is also solved numerically to evaluate the three-dimensional $J$ integral at the onset of breakdown in the specimen. It is shown that the $J$ integral for the channel at the initiation of a new channel can be obtained from the solution of the initial channel. The length and radius of the initial channel affect strongly the $J$ integral. The $J$ integral for the channel is directly related to the energy released per unit length of the channel, as a result of channel growth. The $J$ integral is used as a breakdown parameter that governs the breakdown process. The critical $J$ integral at the onset of breakdown is obtained.
4.1 Introduction

The purpose of present chapter is to investigate tubular channel growth in a piezoelectric material with a conductive channel. Breakdown tests are performed on PZT807 samples with cylindrical bar shapes under purely electrical loading. It is experimentally observed that dielectric breakdown occurs via the formation of tubular channels, and the new tubular channel propagates in a straight direction through the specimen. The three-dimensional $J$ integral for a tubular channel is used as a criterion for dielectric breakdown failure. The $J$ integral at the onset of breakdown is calculated numerically through finite element analysis. The critical $J$ integrals at the onset of breakdown are obtained.

4.2 The Experiment

4.2.1 Experimental Procedure

The material used in this study is a poled PZT807 ceramic (Morgan Electro Ceramics, UK), which withstands high levels of electric field and mechanical stress. The geometry of the specimens and the electrical loading conditions are schematically shown in Fig. 4.1. The specimens had the shape of a cylindrical bar with radius $R=2.5\text{mm}$ and length $L=10\text{mm}$. A small channel was perpendicularly drilled at the center of one end-face of each cylindrical bar sample. The length of the initial channel,
a, was either 5mm or 7mm. The radius of the initial channel, \( r_0 \), ranged between 0.3mm and 0.7mm in increments of 0.1mm. After the drilling, the samples were cleaned in distilled water. The drilled holes were filled with silver paint to create the conductive channel. The silver paint was colloidal silver liquid (PucoTech, Korea) and solidified at room temperature in 24 hours. Both end-faces of samples were painted to render them as electrodes. Both electrodes were connected with a high voltage supplier with a maximum of 50kV (Glassman High Voltage, Japan).

The experiment on dielectric breakdown was carried out according to the ASTM standard D3755-97 (2004). The specimens were tested under purely electrical loading. To investigate the effect of poling directions on breakdown behavior, samples with either a positive or a negative poling direction were used, as shown in Fig. 4.1. The positive poling direction was the same as that of the applied electric field, while the negative poling direction was opposite to that of the applied electric field. Regardless of the poling directions in the samples, that electrode surface, from which the initial tubular channel originated, was connected to the cathode of the high voltage supply, while the opposite end-face was connected to the anode. The samples were immersed in silicon oil to prevent flashover and to minimize the effect of surface discharges throughout the course of the experiment until breakdown. The voltage was increased at a uniform rate of 500V/s from zero V until breakdown. The breakdown voltages were recorded for the samples.

4.2.2 Experimental Results

The electric field is intensified around the head of the initial channel that is embedded in a poled PZT807. Narrow tubular channels were initiated from the head of the initial channel. Fig. 4.2 shows optical microscopic pictures of the new tubular channels that emanated from the head of the initial channel during dielectric breakdown; such channels propagated in a straight direction through the specimen. The
radius of the new tubular channel was approximately 30μm for all the specimens, regardless of the poling direction. Fig. 4.3 shows scanning electron micrographs of an undamaged region and a new channel surface in the specimen following dielectric breakdown. It is seen from Fig. 4.3 that the growth of the new tubular channel is accompanied with the melt of the material during dielectric breakdown. As Fig. 4.4 shows, almost all specimens broke transversely into two parts, beginning from the head of the initial channel. For specimens with a positive poling direction, the profile of the fracture surface was parabolic in shape, and the part with the initial tubular channel was cup-shaped. For most specimens with a negative poling direction, the profile of the fracture surface was approximately elliptical in shape, and the part with the initial tubular channel was oblate cup-shaped; for very few specimens with a negative poling direction, the shape of the fracture surface was relatively flat.

The voltages at the occurrence of dielectric breakdown were recorded by using the high voltage supplier during the test. The breakdown voltages, \( V_{DB} \), are plotted in Figs. 4.5 and 4.6 for samples with positive and negative poling directions, respectively. The breakdown voltages are scattered across the piezoelectric ceramics. The scatter may be attributed to differences in head shape of the initial channel and in the pattern of channel formation, when there is at least one channel. The length of the initial channel influences the breakdown voltage, which is less sensitive to the radius of the initial channel.

4.3 Numerical Analysis

In this study, we employ the three-dimensional \( J \) integral for a tubular channel as a criterion of failure that is due to dielectric breakdown. The breakdown that is due to the propagation of a channel occurs at a critical value of the \( J \) integral, \( J_c \), which is a measure of breakdown resistance.

For calculating the \( J \) integral at the onset of a new channel, consider a new tubular
channel with radius \( c \) and length \( b \) that emanates from the head of the initial channel with radius \( c_0 \) and length \( a \), as shown in Fig. 4.7. The head-shapes of the initial channel and the new channel are respectively assumed to be hemispherical and spherical caps with the same radius, \( c_0 \). In this problem, the three-dimensional \( J \) integral (2.8) that is determined in Section 2.4 is used to analyze the dielectric breakdown problem of a poled PZT 807 ceramic with a conductive channel. The three-dimensional \( J \) integral for a conductive channel is given by

\[
J = \int_{S_0} (Wn_i - t_j u_{j,1} + n_j D_j E_i) dS .
\] (2.8)

The longitudinal axis of the channel coincides with the \( x_1 \)-axis of the system of Cartesian coordinates. Assuming that mechanical tractions on the conductive channel are free, it can be shown that Eq. (2.8) can be rewritten as

\[
J = \int_{S_h} (Wn_i + n_j D_j E_i) dS ,
\] (4.1)

where \( S_h \) is the head-surface of the new channel.

The PZT807 piezoelectric material with a poling direction that is along the positive \( x_1 \)-axis is transversely isotropic, and the \( x_2x_3 \) plane coincides with the isotropic plane. The relations that constitute the PZT807 ceramic are written below in matrix form.

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{23} \\
\sigma_{31} \\
\sigma_{12}
\end{bmatrix} = \begin{bmatrix}
C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\
C_{12} & C_{33} & C_{33} & 0 & 0 & 0 \\
C_{12} & C_{33} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{C_{33} - C_{22}}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{55}
\end{bmatrix} \begin{bmatrix}
\gamma_{11} \\
\gamma_{22} \\
\gamma_{33} \\
2\gamma_{23} \\
2\gamma_{13} \\
2\gamma_{12}
\end{bmatrix} - \begin{bmatrix}
e_{11} & 0 & 0 \\
e_{12} & 0 & 0 \\
e_{12} & 0 & 0 \\
e_{23} & 0 & 0 \\
e_{13} & 0 & 0 \\
e_{12} & 0 & 0
\end{bmatrix} \begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix} ,
\] (4.2)
Here, $\sigma_{ij}$ and $\gamma_{ij}$ are the stresses and strains, respectively. $C_{ij\beta}$ is the elastic stiffness, $e_{ij\beta}$ is the piezoelectric constant and $\varepsilon_{ij}$ is the dielectric constant. The material constants for PZT807 in the positive poling direction are listed in Table 1. The electric enthalpy density for the linear piezoelectric material is given by

$$W = \frac{1}{2} (\sigma_{ij} \gamma_{ij} - D_i E_j).$$

(4.3)

Once the electric field and the elastic field are determined, the $J$ integral for the new channel can be evaluated from Eq. (2). Piezoelectric analyses are carried out with the ABAQUS finite element program for calculating the electric field and the elastic field on the head-surface of the new channel. The eight-node, biquadratic, axisymmetric piezoelectric quadrilateral element, CAX8E, is used in the computations. No difference between the $J$ integral for the channel with length $b$ and that for the case of $b=0$ is observed as $b \to 0^+$. Owing to the continuity of the $J$ integral at $b=0$, a simple method can be used to obtain the integral for the channel at the onset of breakdown from just the solution of the initial channel ($b=0$) – without having to solve the case when $b>0$. The $J$ integrals at the initiation of a new tubular channel with $c=30 \mu$m and $b=0^+$ are numerically calculated for all the samples used in this study to obtain the breakdown resistance.

### 4.4 Breakdown Resistance and Discussion

When the $J$ integral reaches a critical value, $J_c$, breakdown occurs that is due to the propagation of a channel. The critical $J$ integrals at the onset of breakdown for the cases
of positive and negative poling directions are shown in Figs. 4.8 and 4.9, respectively. The breakdown resistance, $J_c$, is almost independent of the length of the initial channel, while $J_c$ slightly decreases as the radius of the initial channel increases. The breakdown resistance is about 0.0125 J/m and 0.0108 J/m for the samples with positive and negative poling directions, respectively. This result implies that the $J$ integral can be used as a criterion for breakdown failure in a piezoelectric material with a channel.

There is considerable scatter in the measured values of the critical $J$ integral. The radius of the narrow channel that is created during the breakdown process plays an important role in determining the critical $J$ integral. The breakdown resistance of the piezoelectric material is increased or decreased depending on the new channel radius $c$. The radius of the new tubular channel is about 30 μm for most specimens, and a few samples have a radius in the range of 25 μm - 35 μm. The PZT807 ceramic is endowed with high stiffness, so that the micro-drill is prone to wear during drilling process. The abrasion of the drill affects the shape of the head of the initial channel, which varies from that of a hemisphere to that of an oblate spheroid, as the radius of the drill decreases. The electric field distribution on the surface of the channel head depends upon the shape of the channel head. Thus, the shape of the channel head affects the value of the $J$ integral.

Regardless of the direction of poling, almost all the specimens broke transversely into two parts, beginning from the head of the initial channel. The linear piezoelectric theory predicts that stresses in a sample under a positive electric field have the opposite sign to those under a negative electric field – which is at variance with our experimental observation of fracture behavior. Dielectric breakdown results in channel formation due to melt of the material. During the formation of the channel, the temperature around the channel is rapidly increased, which results in thermal stresses in the sample. A transient, thermo-piezoelectric analysis is required to yield a theoretical explanation for the fracture behavior of the sample during breakdown.
4.5 Summary

Dielectric breakdown in a poled PZT807 ceramic with a conductive channel is investigated. Specimens with the shape of a cylindrical bar are used in tests of dielectric breakdown under purely electrical loading. To investigate the effect of poling directions on breakdown behavior, samples with either a positive or a negative poling direction are used in tests of breakdown. Narrow tubular channels are initiated from the surface of the head of the initial channel during dielectric breakdown, and propagated in a straight direction through the specimen. The radius of the new tubular channel is nearly constant for all of the specimens, regardless of the direction of the applied electric field. The growth of the new tubular channel is accompanied with melt of the material during dielectric breakdown. Almost all specimens broke transversely into two parts beginning from the head of the initial channel. Breakdown voltages were experimentally measured through the high voltage supplier during the test of dielectric breakdown. The length of the initial channel is found to influence the breakdown voltage.

The three-dimensional $J$ integral for a tubular channel is introduced as a criterion of breakdown failure. The breakdown due to the propagation of a channel occurs at a critical $J$ value, which is a measure of breakdown resistance. Piezoelectric analyses are carried out with the finite element program to calculate the electric field and the elastic field. The $J$ integral at the onset of breakdown is calculated numerically for all the samples. For the cases of positive and negative poling directions, the critical $J$ integral at the onset of breakdown is obtained. The breakdown resistance is almost independent of the length of the initial channel, while it slightly decreases as the radius of the initial channel increases. The breakdown resistance is insensitive to the direction of the applied electric field.
CHAPTER 5

ELECTRICAL FAILURE OF PIEZOELECTRIC CERAMICS WITH A CONDUCTIVE CRACK UNDER ELECTRIC FIELDS

5.1 Introduction

In present chapter, the failure behavior under purely electric loading of piezoelectric ceramics with a conductive crack is investigated. The study lies on the experimental observation made by 박재연 (2003). According to his observation, two failure phenomena, fracture and dielectric breakdown, are occurred. And the dielectric breakdown occurs via the formation of tubular channels and that the radii of the tubular channels are almost independent of the poling direction and the direction of electric loading. The critical $J$ integrals at the onset of both fracture and breakdown are calculated numerically via finite element analysis. The effects of both the direction of the electric field and the poling direction on both fracture and breakdown resistance are discussed. The possibilities of fracture and dielectric breakdown are also discussed.

5.2 Numerical Analysis

Finite element analysis is conducted to investigate the fracture and dielectric breakdown of piezoelectric ceramics with conducting cracks. In particular, we discuss how the crack or tubular channel begins to grow in piezoelectric ceramics. This analysis is based on the work carried out by 박재연 (2003), in which the effects of both electric field direction and poling direction are quantified with a experimental approach. The electrical failure experiments were performed on a plate shape PZT DE-DL piezoelectric ceramic with a conductive crack as shown in Fig. 5.1.
To understand the growth of the crack and of the tubular channel, we carry out piezoelectric analysis with the commercial software, ABAQUS. We adopt the eight-node, biquadratic, plane strain, piezoelectric quadrilateral element, CPE8E, in both 2-D crack problems and 3-D breakdown problems. The material constants of PZT DE-DL that are used in the numerical analysis are listed in Table 2. Fig. 5.2 shows the mesh configuration and the distribution of the electric field around the tip of the conducting crack. Both the size of the FE model and the applied boundary conditions are comparable to those that prevail in the experimental conditions. It is evident from Fig. 5.2 that the electric field is concentrated and intensified around the crack tip. This concentration and intensification can promote the initiation of the crack or tubular channel at the crack tip. This is in agreement with the experimental observations.

The three-dimensional $J$ integral, which has been introduced in Section 2.4, is employed to establish a criterion of failure that is due to dielectric breakdown. The breakdown that is induced by the formation of a tubular channel occurs at a critical value of the $J$ integral, $J_c^*$, which is the measure of breakdown resistance. In present chapter, the superscript (*) indicates the three-dimensional $J$ integral used as a failure criterion for a dielectric breakdown problem. In addition, the two-dimensional, critical $J$ integral for a crack, $J_c$, is also evaluated as a measure of the fracture resistance. The two-dimensional $J$ integral for a crack in a piezoelectric material is also introduced in Section 2.4.

In the present work, we study the crack growth and the channel formation. Consider the two-dimensional, $J$ integral, which is introduced in Section 2.4 and is defined as (Pak, 1990)

$$J = \int_\Gamma (Wn_1 - t_j u_{j,1} + n_j D_j E_1) d\Gamma .$$

(2.7)

Here, $n_j$ is the unit outward vector that is normal to an arbitrary contour $\Gamma$ that encloses the tip of the conducting crack, as shown in Fig. 5.3. The longitudinal axis of
the crack lies along the $x_1$-axis of the system of Cartesian coordinates. We evaluate the three-dimensional $J$ integrals as of the onset of the channel formation. We introduce the three-dimensional $J$ integral, which is defined as

$$J^* = \int_S (Wn_i - t_ju_{j,i} + n_jD_jE_i) dS.$$  (5.1)

Here, $S$ is an arbitrary surface that encloses the tip of the tubular channel with radius $c$ and length $b$ as shown in Fig. 5.4.

The piezoelectric material with a poling direction of $\psi = 0$ that is along the positive $x_1$-axis is transversely isotropic, and the $x_2x_3$ plane coincides with the isotropic plane. The relations that constitute the piezoelectric ceramic are written below in matrix form.

$$\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{23} \\
\sigma_{31} \\
\sigma_{12}
\end{bmatrix} = \begin{bmatrix}
C_{11} & C_{12} & C_{13} \\
C_{12} & C_{11} & C_{13} \\
C_{13} & C_{12} & C_{11} \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\gamma_{11} \\
\gamma_{22} \\
\gamma_{33} \\
2\gamma_{23} \\
2\gamma_{13} \\
2\gamma_{12}
\end{bmatrix} - \begin{bmatrix}
\varepsilon_{33} & 0 & 0 \\
0 & \varepsilon_{11} & 0 \\
0 & 0 & \varepsilon_{15}
\end{bmatrix} \begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix}. (5.2)

$$\begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix} = \begin{bmatrix}
e_{33} & e_{31} & e_{31} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & e_{15} \\
0 & 0 & 0 & 0 & 0 & e_{15}
\end{bmatrix} + \begin{bmatrix}
\gamma_{11} \\
\gamma_{22} \\
\gamma_{33} \\
2\gamma_{23} \\
2\gamma_{13} \\
2\gamma_{12}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{33} & 0 & 0 \\
0 & \varepsilon_{11} & 0 \\
0 & 0 & \varepsilon_{11}
\end{bmatrix} \begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix}. (5.3)

Here, $\sigma_{ij}$ and $\gamma_{ij}$ are the stresses and strains, respectively. $C_{ijkl}$ is the elastic
stiffness, $e_{ij}$ is the piezoelectric constant, and $\varepsilon_{ij}$ is the dielectric constant. The electric enthalpy density for the linear piezoelectric material is given by:

$$W = \frac{1}{2}(\sigma_{ij}\nu_{ij} - D_{ij}E_i).$$

We can also easily express the constitutive relations of the piezoelectric material with a poling direction of $\psi = \pi/2$.

To obtain the fracture and breakdown resistances for the four cases described above, the two and three dimensional $J$ integrals are numerically calculated with $c=30\mu m$ and $c_0=75\mu m$ for all the samples that are used in the present work.

5.3 Numerical Results and Discussion

We can define the critical $J$ integral of cracks on the basis that a new crack can propagate if the two-dimensional $J$ integral reaches a critical value, $J_c$. The critical $J$ integral of tubular channels that are caused by dielectric breakdown can also be defined on the grounds that a tubular channel can propagate if the three-dimensional $J$ integral of the tubular channel reaches a critical value, $J_c^*$. Fig. 5.5 graphs the calculated $J_c$ for four cases. It is seen from Fig. 5.5 that regardless of the poling direction, the critical $J$ integral, $J_c$, is much higher under a negative electric field than under a positive electric field. In the analysis of dielectric breakdown, we also derived a similar trend, as shown in Fig. 5.6. Here $b=0^\circ$ has been used in the evaluation of the $J^*$ integral at the onset of channel growth. This implies that a positive electric field can decrease the fracture and breakdown resistance of piezoelectric materials, whereas a negative electric field can increase the fracture and breakdown resistance of piezoelectric materials. In addition, from Figs. 5.5 and 5.6, we can confirm that the poling direction can also affect the $J$ integral of piezoelectric materials; thus, under the same electric field, the $J$ integral of piezoelectric materials with a poling direction of $\psi = \pi/2$ is higher than the $J$ integral of piezoelectric materials with a poling direction of $\psi = 0$. Hence, the poling direction also can affect the fracture and breakdown resistance of piezoelectric materials. Fracture or dielectric breakdown occurs if the applied $J$ integral reaches a critical value,
viz., $J_c$ or $J'_c$. In addition, we can predict the possibility of fracture or breakdown by comparing the values of $J/J_c$ and $J^*/J'_c$. If $J/J_c$ is higher than $J^*/J'_c$, fracture occurs prior to dielectric breakdown, and vice versa. However, some problems still remain to be considered in future research. It is observed in experiments that dielectric breakdown occurs via the formation of fine tubular channels, which melts down the specimen. A large thermal energy may be released from the tip of the conducting crack at the onset of dielectric breakdown due to the initiation of the tubular channel. This thermal energy may result in a thermal stress that affects the breakdown resistance of the piezoelectric material. Therefore, thermo-piezoelectric analysis is required for obtaining a theoretical explanation for the dielectric breakdown of the sample. Furthermore, domain-switching is also to be considered in future studies. Owing to the intensification of the electric field around the tip of the conducting crack, the electric field will easily exceed a critical value before dielectric breakdown occurs. This forms an electric saturation for inducing domain-switching around the tip of the conducting crack. Domain-switching promotes the redistribution of the strain and stress around the tip of the conducting crack, and this redistribution may require modification of the solution of the electric fields.

5.4 Summary

The failure behavior of piezoelectric ceramics with a conducting crack that are subject to purely electric loading is investigated. The study lies on the experimental observation made by 박재연 (2003). According to his observation, under electrical loads, specimens fail in one of two modes: fracture accompanied with dielectric discharging and the formation of tubular channels without fracture. The $J$ integral that is widely used in fracture mechanics is introduced for the dielectric breakdown problem. To solve the problems of the growth of the crack and of the tubular channel from the initial conducting crack, piezoelectric analyses are conducted for evaluating the two and
three dimensional $J$ integrals at the onset of fracture and breakdown in piezoelectric ceramics, respectively. The critical $J$ integrals at the onset of fracture or breakdown are obtained. A positive electric field can decrease the breakdown resistance of piezoelectric materials, whereas a negative electric field can increase the breakdown resistance of piezoelectric materials. The poling direction also can affect the breakdown resistance of piezoelectric materials.
CHAPTER 6

FAILURE BEHAVIOR OF A PIEZOELECTRIC BIMATERIAL WITH AN INTERFACIAL ELECTRODE

6.1 Introduction

In present chapter, Fracture behavior of a piezoelectric bimaterial with an interfacial electrode subjected to purely electric loading is investigated. Plate shape specimens with interfacial electrodes are used for electrical fracture tests. Cracks emanating from the tip of interfacial electrodes are experimentally observed. The mutual integral, which has the conservation property, is applied to obtain electric field intensity factors due to purely electrical loading for interfacial electrodes between dissimilar piezoelectric materials. The critical electric field intensity factors at the onset of crack growth are obtained. The critical energy release rates at the onset of crack growth are also calculated.

6.2 Experimental Procedures

The material used in the electrical failure test is the hard lead zirconate titanate fabricated by PZT C-201 and PZT C-3 (Fuji Ceramics Co., Ltd., Japan) through perfect bonding. In addition, the thin electrode lies on the interface between PZT C-201 and PZT C-3. The geometry of the specimens and the electrical loading conditions are schematically shown in Fig. 6.1.

All of the specimens had rectangular plate shape with $10 \times 10 \times 3 \text{mm}^3$ size. The lengths of the interfacial electrodes, $a$ were 5mm and 7mm, respectively. The both $10 \times 3 \text{mm}^2$ sides of the specimens, which are perpendicular to the interface, were
painted by using silver paint to create both electrodes. We used the high voltage supplier with maximum voltage of 50kV (Glassman High Voltage, INC.) to generate DC electric fields on the specimens. All of the experiments were performed by immersing the test samples in silicone oil to minimize the effects of surface discharges prior to electrical failure. All the failure tests are followed the standard of ASTM D3755-97 (2004), and are conducted at room temperature.

6.3 Experimental Results

Under electrical loading condition, the specimens show two kinds of the failure modes, that is fracture by crack propagation with electrical discharge and dielectric breakdown by growth of conducting path. Most of the specimens with electrode of length of 5mm are fractured by growth of the crack, whereas, most of the specimens with electrode of length of 7mm are damaged by growth of the tubular channel. In addition, the fracture mode is separated two kinds of modes as well, that is crack kinking out of the interface plane and crack growth along the interface as shown in Fig. 6.2.

The failure voltages were recorded by using high voltage supplier during electrical failure. The electrical failure voltages obtained from the experiments are shown in Fig. 6.3. The electrical failure voltages scattered in piezoelectric bimaterials. This scattering may be attributed to the mismatch of the electromechanical properties of the two dissimilar materials and brittleness of the materials. It is observed that the magnitude of the electrical failure voltages strongly depends on the length of the interface electrode.

6.4 Complex Variable Method

6.4.1 Near Tip Stress and Electric Fields
Consider a conducting crack embedded between two dissimilar anisotropic piezoelectric ceramics as shown in Fig. 6.4. Material 1 and 2 occupy the regions above and below the $x_1$-axis, respectively. The crack tip is located at the origin, and the crack surfaces are electrically conductive and traction-free. About the general solutions of displacement, electric potential, stress and electric field, we have presented in Section 2.5. The vector of stress and electric field intensity factors which uniquely characterize the singular field near the tip of conducting crack can be defined by

$$k = \lim_{x_i \to 0^+} \sqrt[2]{2\pi x_i} Y(x_i^{+i\kappa},x_i^{-i\kappa}) r(x_i),$$

(6.1)

Here,

$$
\varepsilon = \frac{1}{2\pi} \ln \frac{1+\eta}{1-\eta}, \quad \kappa = \frac{1}{2\pi} \ln \frac{1+\omega}{1-\omega}, \quad \eta = [\{(1/4) tr(\beta^2)^2 - \|\beta\|_{1/2}^2 - (1/4) tr(\beta^2)\}_{1/2}], \quad \\
\omega = [\{-[(1/4) tr(\beta^2)^2 - \|\beta\|_{1/2}^2 - (1/4) tr(\beta^2)\}_{1/2}], \quad \beta = (L^{(1)}-1 + L^{(2)}-1)^{-1}(M^{(1)} - M^{(2)}),

(6.2)

where $tr$ represents the trace of a matrix. The matrix function $Y(\xi(z),\zeta(z))$ is expressed explicitly in terms of the real bimaterial matrix $\beta$, as

$$Y(\xi(z),\zeta(z)) = \frac{1}{2} \left\{ -\frac{\omega^2}{\eta^2 - \omega^2} \left[ \xi(z) + \zeta(z) \right] + \frac{\omega^2}{\eta^2 - \omega^2} \left[ \zeta(z) + \xi(z) \right] \right\} I

+ \frac{1}{2} \left\{ -i \frac{\omega^2}{\eta^2 - \omega^2} \left[ \xi(z) - \zeta(z) \right] + \frac{\eta \omega^2}{\eta^2 - \omega^2} \left[ \zeta(z) - \xi(z) \right] \right\} \beta

+ \frac{1}{2} \left\{ \frac{1}{\eta^2 - \omega^2} \left[ \xi(z) + \zeta(z) \right] + \frac{i \omega^2}{\eta^2 - \omega^2} \left[ \zeta(z) + \xi(z) \right] \right\} \beta^2

- \frac{1}{2} \left\{ i \frac{\omega^2}{\eta^2 - \omega^2} \left[ \xi(z) - \zeta(z) \right] - \frac{i}{\omega \eta^2 - \omega^2} \left[ \zeta(z) - \xi(z) \right] \right\} \beta^3
\]
where \( \xi(z) \) and \( \zeta(z) \) are arbitrary functions of \( z \). \( k = (K_H K_I K_M K_E)^T \). In terms of \( k \), the analytic functions generating the singular part of the interface stress and electric field can be expressed as (Beom and Atluri, 2002)

\[
B^{(1)} f^{(1)}(z) = \frac{1}{2 \sqrt{2\pi}} (I + i\beta) Y(z^{1/2}, z^{1/2}) k ,
\]

\[
B^{(2)} f^{(2)}(z) = \frac{1}{2 \sqrt{2\pi}} (I - i\beta) Y(z^{1/2}, z^{1/2}) k .
\] (6.4)

6.4.2 M Integral

The path-independent \( J \) integral for a linear piezoelectric material with a crack is proposed by Pak (1990), and is introduced in Section 2.4. The \( J \) integral (2.7) can be rewritten as

\[
J^{(v; \Gamma)} = \frac{1}{4} \int_{\Gamma} W n_i - t_j v_{j,i} d\Gamma ,
\] (6.5)

where \( W = \frac{1}{2} \Sigma_{ij} v_{ij} \) is the electric enthalpy density, \( n_i \) is the unit outward vector normal to an arbitrary contour \( \Gamma \) enclosing the crack tip, \( t_j = n_i \Sigma_{ij} \) is the surface traction and the surface electric field. The \( J \) integral can be represented in terms of the near tip fields derived above employing the relation between the \( J \) integral and the intensity factors as follows:

\[
J^{(v; \Gamma)} = \frac{1}{4} k^T U^{-1} k .
\] (6.6)

where \( U^{-1} = (L^{(1)} + L^{(2)}) (I + \beta^2) \).
Consider two independent equilibrium states of a piezoelectrically deformed bimaterial body, with each displacement and electric potential being indicated by \( \mathbf{v} \) and \( \mathbf{\tilde{v}} \), respectively. The mutual integral for the two states is represented by

\[
M \{ \mathbf{v}, \mathbf{\tilde{v}}; \Gamma \} = J \{ \mathbf{v} + \mathbf{\tilde{v}}; \Gamma \} - J \{ \mathbf{v}; \Gamma \} - J \{ \mathbf{\tilde{v}}; \Gamma \}.
\]

(6.7)

The mutual integral also satisfies the conservation law same to the \( J \) integral. Individual stress and electric field intensity factors for the equilibrium state \( \mathbf{v} \) can be calculated by using the mutual integral, if the auxiliary solution, which is obtained for equilibrium state \( \mathbf{\tilde{v}} \), is known.

6.4.3 Intensity Factors

Consider a semi-infinite conducting crack embedded in two dissimilar anisotropic piezoelectric materials as shown in Fig. 6.4. The auxiliary solutions for material 1 and 2 are given by

\[
\tilde{f}^{J(1)}(z) = \frac{1}{2\sqrt{2\pi}} \frac{B^{(1)}}{B^{(2)-1}} (I + i\beta) Y(z^{ie}, z^{ik}) \hat{e}^J, \\
\tilde{f}^{J(2)}(z) = \frac{1}{2\sqrt{2\pi}} \frac{B^{(2)}}{B^{(1)}} (I - i\beta) Y(z^{ie}, z^{ik}) \hat{e}^J,
\]

(6.8)

where \( \hat{e}^J (J = 1, 2, 3, 4) \) is the base vector with the component \( \hat{e}^J_M = \delta_{JM} \) and \( \delta_{JM} \) is the Kronecker delta. From (6.6)-(6.8), the stress and electric field intensity factors for the equilibrium state \( \mathbf{v} \) can be obtained as

\[
k_M = 2U_{MJ} M \{ \mathbf{v}, \mathbf{\tilde{v}}; \Gamma \}.
\]

(6.9)

6.5 Numerical Analysis
6.5.1 Numerical Verification

Finite element analysis is conducted to numerically verify the validity of solution of intensity factors (6.9). The piezoelectric analysis is carried out with commercial program, ABAQUS. We adopted the eight-node, biquadratic, plane strain, piezoelectric quadrilateral, reduced integration element, CPE8RE. Fig. 6.5 shows the finite element model of a semi-infinite electrode embedded between two dissimilar anisotropic piezoelectric materials. The remote field at infinity is assumed to be the near-tip field of electric potential.

\[
\phi = -2 \text{Re} \left[ \sum_{M=1}^{4} B_{AM} f_M(z_M) \right]. \tag{6.10}
\]

The material 1 has a poling direction along the positive \( x_2 \)-axis, and material 2 has a poling direction along the negative \( x_2 \)-axis. The relations that constitute the piezoelectric ceramic are written below in matrix form for materials 1 and 2.
\[
\begin{align*}
\begin{bmatrix}
\sigma_{11}^{(i)} \\
\sigma_{12}^{(i)} \\
\sigma_{22}^{(i)} \\
\sigma_{33}^{(i)} \\
\sigma_{12}^{(i)} \\
\sigma_{13}^{(i)} \\
\sigma_{23}^{(i)} \\
\sigma_{11}^{(i)}
\end{bmatrix} &= 
\begin{bmatrix}
C_{11}^{(i)} & C_{12}^{(i)} & C_{13}^{(i)} & 0 & 0 & 0 \\
C_{12}^{(i)} & C_{22}^{(i)} & C_{23}^{(i)} & 0 & 0 & 0 \\
C_{13}^{(i)} & C_{23}^{(i)} & C_{33}^{(i)} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44}^{(i)} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{11}^{(i)} - C_{12}^{(i)} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{44}^{(i)} \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\gamma_{11}^{(i)} \\
\gamma_{12}^{(i)} \\
\gamma_{13}^{(i)} \\
\gamma_{22}^{(i)} \\
\gamma_{33}^{(i)} \\
\gamma_{44}^{(i)} \\
\gamma_{12}^{(i)} \\
\end{bmatrix},
\end{align*}
\]

(6.11)

\[
\begin{align*}
\begin{bmatrix}
D_{1}^{(i)} \\
D_{2}^{(i)} \\
D_{3}^{(i)}
\end{bmatrix} &= 
\begin{bmatrix}
D_{1}^{(i)} \\
D_{2}^{(i)} \\
D_{3}^{(i)}
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & e_{15}^{(i)} & e_{22}^{(i)} & e_{23}^{(i)} & e_{33}^{(i)} & e_{15}^{(i)} \\
0 & 0 & 0 & 0 & 0 & 0 & 2\gamma_{23}^{(i)} & 0 & 0 & 0 & \gamma_{22}^{(i)} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\gamma_{13}^{(i)} & 0 & 0 & \gamma_{13}^{(i)} \\
\end{bmatrix}
\begin{bmatrix}
\gamma_{11}^{(i)} \\
\gamma_{12}^{(i)} \\
\gamma_{13}^{(i)} \\
\gamma_{22}^{(i)} \\
\gamma_{33}^{(i)} \\
\gamma_{44}^{(i)} \\
\gamma_{12}^{(i)} \\
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & e_{11}^{(i)} \\
0 & 0 & 0 & 0 & 0 & 0 & e_{22}^{(i)} \\
0 & 0 & 0 & 0 & 0 & 0 & e_{33}^{(i)} \\
\end{bmatrix}
\begin{bmatrix}
E_{1}^{(i)} \\
E_{2}^{(i)} \\
E_{3}^{(i)}
\end{bmatrix}.
\end{align*}
\]

(6.12)
Two piezoelectric ceramics, C-201 and C-3, are used in present verification. The material 1 is C-201 ceramic and the material 2 is C-3 ceramic. The material properties of both C-201 and C-3 are listed in Table 3 and Table 4. The four distinct eigenvalues with a positive imaginary part for material 1 and material 2, which are obtained by solving an eighth-order polynomial, are

\[ p_1^{(1)} = 1.3988328i, \]
\[ p_2^{(1)} = 1.0711465i, \]
\[ p_3^{(1)} = 0.9449580 i, \]
\[ p_4^{(1)} = 1.1272081 i, \]  
(6.15)

and

\[ p_1^{(2)} = 1.7349937 i, \]
\[ p_2^{(2)} = 1.1938434 i, \]
\[ p_3^{(2)} = 0.7317143 i, \]
\[ p_4^{(2)} = 1.2499600 i, \]  
(6.16)

respectively. The matrices \( A^{(1)} \) and \( A^{(2)} \), which represent eigenvectors corresponding to the four eigenvalues given in (6.15) and (6.16), are obtained as

\[
A^{(1)} = \begin{bmatrix}
0.8413464 & -0.3645707i & -0.4801675i & 0 \\
0.5401476i & 0.3042556 & 0.5634774 & 0 \\
0 & 0 & 0 & 1 \\
18.5360741 & 4.7687314i & -3.7494733i & 0
\end{bmatrix},
\]  
(6.17)

\[
A^{(2)} = \begin{bmatrix}
0.8919490 & 0.1203551i & 0.3091267i & 0 \\
0.4039000i & -0.0340910 & -0.5813680 & 0 \\
0 & 0 & 0 & 1 \\
-11.56353012 & 4.6153087i & -3.0444855i & 0
\end{bmatrix},
\]  
(6.18)

respectively. Here, the units used in these computations are: force GN; length m; electrical potential GV. Hence, from these matrices, stress is computed in GPa, electric field in GV/m, electric displacement in C/m², stress intensity factors in GPa√m, electric field intensity factors in GV/√m and energy release rates in GJ/m². From eigenvalues \( p_i^m (m=1,2; i=1,2,3,4) \) and matrices \( A^{(1)} \) and \( A^{(2)} \), we can obtain the corresponding matrices \( B^{(1)} \) and \( B^{(2)} \) as
We can obtain the near-tip field of electrical potential for an asymptotic problem of semi-infinite electrode embedded between two dissimilar anisotropic piezoelectric ceramics using combinations of above matrices.

Finite element analysis is implemented under various applied electric field intensity factors. Making use of (6.9), we can obtain the electric field intensity factors near the tip of electrode numerically. If the obtained electric field intensity factors agree with the applied electric field intensity factors, the verification of validity of (6.9) is completed. The numerical results are listed in Table 5. It is seen from Table 5 that the calculated electric field intensity factors near the tip of electrode very agree with the applied electric field intensity factors.

6.5.2 Critical Electric Field Intensity Factors

The calculation of the critical electric field intensity factors is carried out for each specimen using (6.9), and showed in Fig. 6.6. It is seen from Fig. 6.6 that the critical electric field intensity factor strongly depends on the length of the internal electrode. However, there is a problem that still remains to be considered in future research. It is observed in experiments that there are considerable number of specimens that fractured by interface crack kinked out of the interface plane. Thus, the interfacial crack kinking problem is also required to be considered for the piezoelectric materials.
6.6 Summary

Fracture behavior of a piezoelectric bimaterial with an interfacial electrode subjected to purely electric loading is investigated. Plate shape specimens with interfacial electrodes are used for electrical fracture tests. Cracks emanating from the tip of interfacial electrodes are experimentally observed. Most of the specimens with electrode of length of 5mm are fractured by growth of the crack, whereas, most of the specimens with electrode of length of 7mm are damaged by growth of the tubular channel. In addition, the fracture mode is separated two kinds of modes as well, that is crack kinking out of the interface plane and crack growth along the interface. The mutual integral, which has the conservation property, is applied to obtain electric field intensity factors due to purely electrical loading for interfacial electrodes between dissimilar piezoelectric materials. The critical electric field intensity factors at the onset of crack growth are obtained.
CHAPTER 7

Kinking of a Conducting Path in an Anisotropic Dielectric Bimaterial

7.1 Introduction

The purpose of present study is to analyze kinking of a conducting path between two dissimilar anisotropic dielectric materials under purely electric loading. Dielectric breakdown commonly occurs in the dielectric materials by formation of conducting path under a critical electric loading. The problem is formulated using a linear transformation method. Based on the formulation, the complete electric fields in the anisotropic dielectric materials are easily obtained from the solutions of the isotropic dielectric materials. The electric field intensity factors and energy release rates of kinked conducting path are obtained for various inclination angles of material principal axes. Numerical analysis is conducted to examine the validity of the analytic solution, which is obtained based on the linear transform method, using finite element method. It is seen that the numerical solution is very agree with analytic solution.

7.2 Formulation

7.2.1 Anisotropic Dielectric Material

Consider the generalized two-dimensional electrostatics of an anisotropic dielectric material with a conducting crack. The electric potential depend only on the in-plane coordinates, $x_1$ and $x_2$. The general solution of electric potential for a generalized two-dimensional deformation problem of a linear anisotropic piezoelectric ceramic with a conducting crack has been introduced in Section 2.5. For anisotropic dielectric
materials, the general solution of electric potential can be obtained by replacing all piezoelectric constants with zero in the two-dimensional deformation problem of a piezoelectric material, thus,

\[ \phi = -2 \text{Re} \left[ \frac{i}{\epsilon} f(z) \right], \tag{7.1} \]

where \( \text{Re} \) denotes the real part,

\[ \epsilon = \sqrt{\varepsilon_{11} \varepsilon_{22} - \varepsilon_{12}^2}. \tag{7.2} \]

The electric fields and electric displacements are obtained from (7.1) as

\[ E_i = 2 \text{Re} \left[ \frac{i}{\epsilon} e_i f'(z) \right], \tag{7.3} \]

\[ D_j = 2 \text{Re} \left[ d_j f'(z) \right]. \tag{7.4} \]

Here, prime \( (') \) implies the derivative with respect to the associated arguments, and \( f(z) \) is analytic in their arguments, \( z = x_1 + px_2 \), and \( p = i\lambda + \eta \) is a complex number with positive imaginary part, in which

\[ \lambda = \frac{\sqrt{\varepsilon_{11} \varepsilon_{22} - \varepsilon_{12}^2}}{\varepsilon_{22}}, \]

\[ \eta = -\frac{\varepsilon_{12}}{\varepsilon_{22}}. \tag{7.5} \]

Here, \( \lambda \) and \( \eta \) satisfy the condition of \( |\eta| < \lambda \). The vectors \( \mathbf{e} \) and \( \mathbf{d} \) for the anisotropic dielectric material may be written as \( \mathbf{e} = \{1 \ p\}^T \) and \( \mathbf{d} = \{p \ -1\}^T \). The analytic function \( f(z) \) is determined by boundary conditions.
7.2.2 Linear Transformation Method

For a linearly isotropic dielectric material, the general solutions of electric potential, electric fields and electric displacements (7.1), (7.3) and (7.4) can be modified as follows

\[ \hat{\phi} = -2 \text{Re} \left[ \frac{i}{\hat{\varepsilon}} f(\hat{z}) \right], \quad (7.6) \]

\[ \hat{E}_i = 2 \text{Re} \left[ \frac{i}{\hat{\varepsilon}} \hat{e}_i f'(\hat{z}) \right], \quad (7.7) \]

\[ \hat{D}_j = 2 \text{Re} \left[ \hat{d}_j f'(\hat{z}) \right], \quad (7.8) \]

where, hat (\( ^\wedge \)) implies the quantities denoted in the electrostatic problem for an isotropic dielectric material in present study. Here, \( \hat{\varepsilon} \) is permittivity, and \( f(\hat{z}) \) is analytic in their arguments, \( \hat{z} = \hat{x}_1 + \hat{\rho} \hat{x}_2 \), in which \( \hat{\rho} = i \). The vectors \( \hat{e} \) and \( \hat{d} \) for the isotropic dielectric materials may be written as \( \hat{e} = [1 \ i]^T \) and \( \hat{d} = [i \ -1]^T \).

Next, the linear transformation of the general solution from an anisotropic dielectric material to an isotropic dielectric material is derived. Since \( z = \hat{z} \), the Cartesian coordinates between isotropic materials and anisotropic materials have a relation as follows

\[ \hat{x}_i = L_{ij} x_j, \quad (7.9) \]

where
\[
\mathbf{L} = \begin{bmatrix}
1 & 0 \\
\eta & \lambda
\end{bmatrix},
\]

(7.10)

in which \( \mathbf{x} = \{x_1, x_2\}^T \) and \( \hat{\mathbf{x}} = \{\hat{x}_1, \hat{x}_2\}^T \) are the Cartesian coordinates in an anisotropic material and in an isotropic material, respectively. The permittivity is

\[
\hat{\varepsilon} = \varepsilon,
\]

(7.11)

in an isotropic dielectric material, where \( \varepsilon \) is the dielectric constant of an anisotropic dielectric materials defined in (7.2). The electric potential also is

\[
\phi = \hat{\phi}.
\]

(7.12)

Therefore, the electric fields in an anisotropic dielectric material are obtained from (7.12) as

\[
E_i = -\frac{\partial \hat{\phi}}{\partial x_i}.
\]

(7.13)

Making use of (7.9) and chain rule, the electric fields (7.13) become

\[
E_i = L_{ij} \hat{E}_j.
\]

(7.14)

We can also obtain the electric displacements in an anisotropic dielectric material as

\[
D_i = A_{ij} \hat{D}_j,
\]

(7.15)

where
\[ \Lambda = \begin{bmatrix} \lambda & -\eta \\ 0 & 1 \end{bmatrix}. \] (7.16)

We introduce the cylindrical coordinates \((r, \theta)\) with the origin located at the conducting crack tip in both anisotropic and isotropic dielectric materials as

\[
\begin{aligned}
    x_1 &= r \cos \theta, \quad x_2 = r \sin \theta, \\
    \hat{x}_1 &= \hat{r} \cos \hat{\theta}, \quad \hat{x}_2 = \hat{r} \sin \hat{\theta}.
\end{aligned}
\] (7.17) (7.18)

With (7.17), (7.18) and (7.9), we can easily derive

\[
\hat{r} = \left( \cos \theta + \eta \sin \theta \right)^2 + \left( \lambda \sin \theta \right)^2 \frac{1}{\cos^2 \theta} r, \] (7.19)

\[
\hat{\theta} = \begin{cases} 
\cos^{-1} \left[ \frac{\cos \theta + \eta \sin \theta}{\left( \cos \theta + \eta \sin \theta \right)^2 + \left( \lambda \sin \theta \right)^2} \right] & \text{for } 0 \leq \theta < \pi \\
-\pi + \cos^{-1} \left[ -\frac{\left( \cos \theta + \eta \sin \theta \right)}{\left( \cos \theta + \eta \sin \theta \right)^2 + \left( \lambda \sin \theta \right)^2} \right] & \text{for } -\pi < \theta < 0
\end{cases}.
\] (7.20)

Transform the Cartesian components of electric fields and electric displacements into cylindrical components in both anisotropic and isotropic materials as

\[
\begin{bmatrix} E_r \\ E_\theta \end{bmatrix} = \Omega \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}, \quad \begin{bmatrix} D_r \\ D_\theta \end{bmatrix} = \Omega \begin{bmatrix} D_1 \\ D_2 \end{bmatrix},
\] (7.21)

and

\[
\begin{bmatrix} \hat{E}_r \\ \hat{E}_\theta \end{bmatrix} = \hat{\Omega} \begin{bmatrix} \hat{E}_1 \\ \hat{E}_2 \end{bmatrix}, \quad \begin{bmatrix} \hat{D}_r \\ \hat{D}_\theta \end{bmatrix} = \hat{\Omega} \begin{bmatrix} \hat{D}_1 \\ \hat{D}_2 \end{bmatrix},
\] (7.22)
respectively. Here,

\[
\Omega = \begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix}, \quad \Omega = \begin{bmatrix}
\cos \hat{\theta} & \sin \hat{\theta} \\
-\sin \hat{\theta} & \cos \hat{\theta}
\end{bmatrix}.
\] 

(7.23)

Substitutions of (7.21) and (7.22) into (7.14) and (7.15) and using (2.7) yields

\[
\begin{bmatrix}
E_r \\
E_\theta
\end{bmatrix} = \mathbf{L}^* \begin{bmatrix}
\hat{E}_r \\
\hat{E}_\theta
\end{bmatrix}, \quad \begin{bmatrix}
D_r \\
D_\theta
\end{bmatrix} = \mathbf{A}^* \begin{bmatrix}
\hat{D}_r \\
\hat{D}_\theta
\end{bmatrix}.
\]

(7.24)

Here,

\[
\mathbf{L}^* = \begin{bmatrix}
\frac{\left(\cos \theta + \eta \sin \theta\right)^2 + (\lambda \sin \theta)^2}{\left(\cos \theta + \eta \sin \theta\right)^2 + (\lambda \sin \theta)^2}^{\frac{1}{2}} & 0 \\
\frac{2\eta \cos 2\theta + (\lambda^2 + \eta^2 - 1)\sin 2\theta}{2\left(\cos \theta + \eta \sin \theta\right)^2 + (\lambda \sin \theta)^2}^{\frac{1}{2}} & \lambda
\end{bmatrix},
\]

\[
\mathbf{A}^* = \begin{bmatrix}
\frac{\lambda}{2\left(\cos \theta + \eta \sin \theta\right)^2 + (\lambda \sin \theta)^2}^{\frac{1}{2}} & \frac{-2\eta \cos 2\theta + (1 - \lambda^2 - \eta^2)\sin 2\theta}{2\left(\cos \theta + \eta \sin \theta\right)^2 + (\lambda \sin \theta)^2}^{\frac{1}{2}} \\
0 & \frac{2\left(\cos \theta + \eta \sin \theta\right)^2 + (\lambda \sin \theta)^2}{\left(\cos \theta + \eta \sin \theta\right)^2 + (\lambda \sin \theta)^2}^{\frac{1}{2}}
\end{bmatrix}.
\]

(7.25)

### 7.3 Isotropic Dielectric Material

In this section, we are concerned with the general solution for the problem of a conducting path emanating from the electrode in an isotropic dielectric material as dielectric breakdown occurs. Dielectric breakdown occurs by formation and growth of
the conducting path in a dielectric material, and this is very similar to the phenomenon of fracture caused by nucleation and growth of the crack in an elastic solid. Therefore, dielectric breakdown under electric loading is the electrical equivalent of fracture under mechanical loading in an elastic solid. Many researchers have successfully treated the dielectric breakdown problem based on the basis of linear elastic fracture mechanics (LEFM) (Garboczi, 1988; Suo, 1993; Zhang et al., 2003; Beom and Kim, 2008, Lin et al., 2009). In this study, we also adopt LEFM on the problem of dielectric breakdown. Consider the asymptotic problem of a conducting path emanating from the electrode surface in a linearly isotropic dielectric material subjected to purely electric loading. This conducting path may naturally function as conducting crack. The tip of conducting path is located at the origin of the coordinate system. Invoke the mathematical analogy between an antiplane deformation problem and electrostatic problem, the general solution for the problem of a conducting path grows in an isotropic dielectric material which satisfy the equilibrium equation is expressed with correspondence (Suo, 1993)

\[
\begin{align*}
\begin{bmatrix} \psi \\ \hat{u}_3 \end{bmatrix} &= 2 \text{Re} \begin{bmatrix} f(\hat{z}) \\ g(\hat{z}) \end{bmatrix}, \\
\begin{bmatrix} \hat{E}_1 \\ \hat{\sigma}_{32} \end{bmatrix} &= 2 \text{Re} \begin{bmatrix} \frac{i}{\hat{\epsilon}} f'(\hat{z}) \\ \hat{\mu} g'(\hat{z}) \end{bmatrix}, \\
\begin{bmatrix} \hat{E}_2 \\ -\hat{\sigma}_{31} \end{bmatrix} &= -2 \text{Re} \begin{bmatrix} \frac{i}{\hat{\epsilon}} f'(\hat{z}) \\ \hat{\mu} g'(\hat{z}) \end{bmatrix}, \\
\begin{bmatrix} \phi \\ \hat{T}_3 \end{bmatrix} &= -2 \text{Re} \begin{bmatrix} \frac{i}{\hat{\epsilon}} f(\hat{z}) \\ \hat{\mu} g'(\hat{z}) \end{bmatrix}.
\end{align*}
\]

(7.26)

Here, $\psi$ is the charge potential, $\hat{u}_3$ is the antiplane displacement, $\hat{\sigma}_{3i}$ are the antiplane shear stresses, where the subscript $i=1, 2, 3$ is the antiplane stress potential. $\hat{\mu}$ is the shear modulus, and $i = \sqrt{-1}$. $f(\hat{z})$ and $g(\hat{z})$ are the analytic functions in
terms of the electric field intensity factor and Mode III stress intensity factor, respectively, and are analytic in their arguments, \( \hat{z} = \hat{x}_1 + i \hat{x}_2 \). Since the thickness of a conducting path progressively decreases to zero, the electric displacement in the direction of \( \hat{x}_2 \) and the electric potential across the conducting path are continuous. The conducting path surface is assumed to be electrically conducting, traction-free and charge-free; these assumptions are represented below.

\[
\begin{align*}
\hat{E}_r(\hat{r}, \pm \pi) &= 0, \quad \hat{r} > 0; \\
\hat{\sigma}_\theta(\hat{r}, \pm \pi) &= 0, \quad \hat{r} > 0; \\
\hat{Q}(\hat{r}, \pm \pi) &= 0, \quad \hat{r} > 0.
\end{align*}
\] (7.27) (7.28) (7.29)

In terms of electric field intensity factor, the analytic function generating the singular part of the electric field can be expressed as

\[
f(\hat{z}) = -i \hat{K}_E \sqrt{\frac{\hat{z}}{2\pi}}.
\] (7.30)

Here, \( \hat{K}_E \) is the electric field intensity factor.

### 7.4 Statement of the problem

#### 7.4.1 Formulation

Consider a two-dimensional asymptotic problem for kinking of a conducting path in dissimilar anisotropic dielectric materials as shown in Fig. 7.1. The semi-infinite interface electrode lies along the negative \( x_1 \)-axis, and a conducting path emanated from the edge of electrode. The conducting path has the length of \( b \) and kinked out of the interface with angle \( \omega \) to the positive \( x_1 \)-axis. Materials 1 and 2 occupy the regions above and below the \( x_1 \)-axis, respectively. Dissimilar anisotropic dielectric
materials are perfectly bonded with each other on the interface lies along the positive \( x_1 \)-axis. Since the stiffness of the electrode is much smaller than the stiffness of the dielectric material, and the thickness of electrode is very thin, the electrode may behave as a pre-conductive crack. Therefore, this problem is similar to the problem of the interfacial crack kinking in an anisotropic bimaterial.

In this problem, both surfaces of electrode and conducting path are assumed to be traction-free and charge-free; these assumptions are represented as follows

\[
\sigma_y(r, \pm \pi) = 0, \quad r > 0; \\
\sigma_y(r, \pm \omega) = 0, \quad 0 < r < b; \quad (7.31)
\]

\[
E_y(r, \pm \pi) = 0, \quad r > 0; \\
E_y(r, \pm \omega) = 0, \quad 0 < r < b. \quad (7.32)
\]

In addition, the electric potential and electric displacements across the bonded portion of the interface are continuous. Thus,

\[
\phi^{(1)}(r, 0^+) = \phi^{(2)}(r, 0^-), \quad r > 0; \\
D_{o}^{(1)}(r, 0^+) = D_{o}^{(2)}(r, 0^-), \quad r > 0. \quad (7.33)
\]

Here, the superscript (1) and (2) distinguish the two materials. The remote field at infinity is prescribed to be the near-tip field of the electric potential as follows

\[
\phi^{(m)} = -2 \text{Re} \left[ \frac{i}{\varepsilon^{(m)}} f(z^{(m)}) \right] \quad \text{as} \quad z^{(m)} \to \infty, \\
f(z^{(m)}) = -i\varepsilon^{(m)} K_E \sqrt{\frac{z^{(m)}}{2\pi}}, \quad m = 1, 2. \quad (7.34)
\]

Here, \( K_E \) is the electric field intensity factor, and \( z^{(m)} = x_1 + p^{(m)} x_2 \) \((m=1, 2)\), in
which \( p^{(m)} = i\lambda^{(m)} + \eta^{(m)} \) \((m=1, 2)\).

Next, based on the mind of the linear transformation method, we need to transform some geometric parameters from the case when material is dielectrically anisotropic to the case when material is dielectrically isotropic as shown in Fig. 7.1. Consider the case when material is dielectrically isotropic. Making use of linear transformation (7.9), the Cartesian coordinates in an isotropic dielectric bimaterial can be modified as follows

\[
\hat{x}_j = L^{(m)}_{ij}x_j, \\
L^{(m)} = \begin{bmatrix}
\eta^{(m)} & 0 \\
\lambda^{(m)} & \lambda^{(m)}
\end{bmatrix}, \quad m = 1, 2.
\] (7.35)

Making use of the transformation (7.19) and (7.20), the length of conducting path \( \hat{b} \) and the kink angle of conducting path to the positive \( x_1 \)-axis \( \omega \) are given by

\[
\hat{b} = \left( \cos \theta + \eta^{(m)} \sin \theta \right)^2 + \left( \lambda^{(m)} \sin \theta \right)^2 \frac{1}{2} \hat{b}, \quad m = \begin{cases} 
1 & \text{for } \omega > 0 \\
2 & \text{for } \omega < 0
\end{cases},
\] (7.36)

and

\[
\omega = \begin{cases} 
\cos^{-1} \left[ \frac{\cos \omega + \eta^{(1)} \sin \omega}{\sqrt{\left( \cos \omega + \eta^{(1)} \sin \omega \right)^2 + \left( \lambda^{(1)} \sin \omega \right)^2}} \right] & \text{for } 0 \leq \omega < \pi \\
\pi + \cos^{-1} \left[ \frac{-\left( \cos \omega + \eta^{(2)} \sin \omega \right)}{\sqrt{\left( \cos \omega + \eta^{(2)} \sin \omega \right)^2 + \left( \lambda^{(2)} \sin \omega \right)^2}} \right] & \text{for } -\pi < \omega < 0
\end{cases},
\] (7.37)

respectively. The boundary conditions (7.31)-(7.33) can be modified as

\[
\sigma_{ij}(\hat{r}, \pm \pi) = 0, \quad \hat{r} > 0;
\]
\( \hat{\sigma}_{ij}(\hat{r}, \pm \hat{w}) = 0, \ 0 < \hat{r} < \hat{b} ; \) \( \quad (7.38) \)

\( \hat{E}_r(\hat{r}, \pm \pi) = 0, \ \hat{r} > 0 ; \)
\( \hat{E}_r(\hat{r}, \pm \hat{w}) = 0, \ 0 < \hat{r} < \hat{b} ; \) \( \quad (7.39) \)

\( \hat{\phi}^{(1)}(\hat{r}, 0^+) = \hat{\phi}^{(2)}(\hat{r}, 0^-) ; \)
\( \hat{D}_\theta^{(1)}(\hat{r}, 0^+) = \hat{D}_\theta^{(2)}(\hat{r}, 0^-) . \) \( \quad (7.40) \)

The near-tip field of the electric potential is transformed as

\[
\hat{\phi}(m) = -2 \text{Re} \left[ \frac{i}{\hat{z}(m)} f(\hat{z}(m)) \right] \text{ as } \hat{z}(m) \to \infty ,
\]

\[
f(\hat{z}(m)) = -i \hat{\phi}(m) K \sqrt{\frac{\hat{z}(m)}{2\pi}} , \quad m=1, 2 . \quad (7.41)
\]

Making use of the relations (7.12), (7.14) and (7.15), the electric potential, the electric field and the electric displacement are

\[
\phi(m) = \hat{\phi}(m) , \quad (7.42)
\]
\[
E_i(m) = L_{ij}^{(m)} \hat{E}_j^{(m)} , \quad (7.43)
\]
\[
D_j(m) = A_{ij}^{(m)} \hat{D}_i^{(m)} , \quad m=1, 2 , \quad (7.44)
\]

in Cartesian coordinate system. Here,

\[
\Lambda(m) = \begin{bmatrix}
\lambda(m) & -\eta^{(m)} \\
0 & 1
\end{bmatrix} , \quad m=1, 2 . \quad (7.45)
\]

Making use of the relation (7.24), the cylindrical components of electric field and electric displacement in isotropic dielectric bimaterial are given by
Here,

\[
\left\{ \begin{array}{c} E_r \\ E_\theta \end{array} \right\} = L^{n(m)} \left\{ \begin{array}{c} \hat{E}_r \\ \hat{E}_\theta \end{array} \right\}, \quad \left\{ \begin{array}{c} D_r \\ D_\theta \end{array} \right\} = \Lambda^{n(m)} \left\{ \begin{array}{c} \hat{D}_r \\ \hat{D}_\theta \end{array} \right\}, \quad m = \begin{cases} 1 & \text{for } \theta > 0 \\ 2 & \text{for } \theta < 0. \end{cases}
\]  

(7.46)

\[
L^{n(m)} = \begin{bmatrix}
\frac{\left(\cos \theta + \eta^{(m)} \sin \theta\right)^2 + \left(\chi^{(m)} \sin \theta\right)^2}{\eta^{(m)} \cos 2\theta + \left(\chi^{(m)} \sin \theta\right)^2 - 1} \sin 2\theta & 0 \\
2\left(\cos \theta + \eta^{(m)} \sin \theta\right)^2 + \left(\chi^{(m)} \sin \theta\right)^2 \chi^{(m)} & \frac{\cos \theta + \eta^{(m)} \sin \theta}{\left(\cos \theta + \eta^{(m)} \sin \theta\right)^2 + \left(\chi^{(m)} \sin \theta\right)^2 \chi^{(m)}}
\end{bmatrix}
\]

\[
\Lambda^{n(m)} = \begin{bmatrix}
\chi^{(m)} & -2\eta^{(m)} \cos 2\theta + \left(1 - \chi^{(m)} \eta^{(m)} \right) \sin 2\theta \\
0 & 2\left(\cos \theta + \eta^{(m)} \sin \theta\right)^2 + \left(\chi^{(m)} \sin \theta\right)^2 \chi^{(m)}
\end{bmatrix}
\]

, \( m = 1, 2 \).

(7.47)

7.4.2 Electric Field Intensity Factor

Choi et al. (1994) have solved the problem of interfacial crack kinking in isotropic elastic bimaterial under antiplane shear, and derived Mode III stress intensity factor for kinked interface crack. Based on the mathematical equivalence between Mode III problem and electrostatic problem and making use of the solution of Mode III stress intensity factor of kinked crack obtained by Choi et al. (1994), we can easily obtain the electric field intensity factor near the tip of kinked conducting path in an isotropic dielectric bimaterial as follows
\[
\hat{K}'_E = \hat{c}(\hat{\omega}, \gamma)\hat{K}_E,
\]  
(7.48)

\[
\hat{c}(\hat{\omega}, \gamma) = \left(\frac{\pi - \hat{\omega}}{\pi + \hat{\omega}}\right)^{\frac{\hat{\omega}}{2\pi}} \prod_{k=1}^{\infty} \frac{1 + \frac{\pi + \hat{\omega}}{2k\pi}}{1 + \frac{1}{2q_k}}, \quad \text{for} \quad 0 \leq \hat{\omega} < \pi.
\]  
(7.49)

Here, \( \gamma = (\hat{\varepsilon}^{(1)} - \hat{\varepsilon}^{(2)}) / (\hat{\varepsilon}^{(2)} + \hat{\varepsilon}^{(1)}) \) is Dundurs parameter, and \( \hat{\varepsilon}^{(1)} \) and \( \hat{\varepsilon}^{(2)} \) are the permittivity of material 1 and 2, respectively. \( q_k \) is the positive root for

\[
\sin\{(\pi + \hat{\omega})q\} + \gamma \sin\{(\pi - \hat{\omega})q\} = 0,
\]  
(7.50)

where \( \hat{\omega} = \hat{\omega}(\omega) \). It is very easy to understanding that for the case of \( -\pi < \hat{\omega} < 0 \),
\( \hat{c}(\hat{\omega}, \gamma) = \hat{c}(-\hat{\omega}, -\gamma) \).

Next, let’s focus our attention on the procedure obtaining the solution of electric field intensity factor at the tip of kinked conducting path in an anisotropic dielectric bimaterial using linear transformation method.

The electric field intensity factor near the tip of kinked conducting path in an isotropic bimaterial as shown in Fig. 7.1(b) may be defined as

\[
\hat{K}'_E = \lim_{r \to a^+ \theta \to \infty} \sqrt{2\pi(r - b)}\hat{E}_r.
\]  
(7.51)

For the anisotropic dielectric bimaterial as shown in Fig. 7.1(a), the electric field intensity factor at the tip of kinked conducting path may be defined as

\[
K'_E = \lim_{r \to a^+ \theta \to \infty} \sqrt{2\pi(r - b)}E_r.
\]  
(7.52)
Making use of (7.19) and (7.36), we can obtain

$$\frac{r-b}{\hat{r}-\hat{b}} = \left[ \cos \theta + \eta^{(m)} \sin \theta \right]^2 + \left[ \lambda^{(m)} \sin \theta \right]^2 \right]^{1/2}, \quad m = \begin{cases} 1 & \text{for } \omega > 0 \\ 2 & \text{for } \omega < 0 \end{cases}, \quad \text{(7.53)}$$

and it is seen from (7.46) that

$$E_r = \left[ \cos \theta + \eta^{(m)} \sin \theta \right]^2 + \left[ \lambda^{(m)} \sin \theta \right]^2 \right]^{1/2} \hat{E}_r, \quad m = \begin{cases} 1 & \text{for } \theta > 0 \\ 2 & \text{for } \theta < 0 \end{cases}. \quad \text{(7.54)}$$

With (7.48), (7.49) and (7.51)-(7.54), we can obtain the electric field intensity factor near the tip of kinked conducting path in an anisotropic material as

$$K_E = c(\omega, \gamma, \lambda^{(m)}, \eta^{(m)}) K_E, \quad \text{(7.55)}$$

$$c(\omega, \gamma, \lambda^{(m)}, \eta^{(m)}) = \left[ \left( \cos \omega + \eta^{(m)} \sin \omega \right)^2 + \left( \lambda^{(m)} \sin \omega \right)^2 \right]^{1/2} c(\omega, \gamma), \quad \text{(7.56)}$$

$$m = \begin{cases} 1 & \text{for } \omega > 0 \\ 2 & \text{for } \omega < 0 \end{cases}.$$

Fig. 7.2. illustrates the normalized electric field intensity factor $c(\omega, \gamma, \lambda^{(m)}, \eta^{(m)})$ as a function of $\omega$ for various combinations of $\lambda^{(1)}, \lambda^{(2)}, \eta^{(1)}$ and $\eta^{(2)}$ when $\gamma = 0.5$. It is seen from Fig. 7.2 that the electric field intensity factor has a discontinuity at $\omega = 0$.

7.4.3 Energy Release Rate

The energy release rate, which includes mechanical and electrical energy during
crack advance, is frequently used as a crack kinking criterion for electro-materials such as dielectrics and piezoelectrics (Qin and Zhang, 2000; Zhu and Yang, 1999; Zhong and Li, 2006; Jeong et al., 2008). For the kinked conducting path in an anisotropic dielectric bimaterial, the energy release rate can be expressed in terms of electric field intensity factor as follows

\[
G = \frac{1}{2} \varepsilon^{(m)} \left( K_E^* \right)^2, \quad m = \begin{cases} 
1 & \text{for } \omega > 0 \\
2 & \text{for } \omega < 0
\end{cases}
\]

(7.57)

where \( \varepsilon^{(m)} = \sqrt{\varepsilon_{11}^{(m)} \varepsilon_{22}^{(m)} - \varepsilon_{12}^{(m)2}} \) is dielectric constant of materials \( m \). The dimensionless form of energy release rate can be rewritten as

\[
G = G_0 \tilde{G}(\gamma, \lambda^{(m)}, \eta^{(m)}), \quad m = \begin{cases} 
1 & \text{for } \omega > 0 \\
2 & \text{for } \omega < 0
\end{cases}
\]

(7.58)

Here, \( G_0 \) is the energy release rate for the case when conducting path advances along the interface. The conducting path may emanate either by straight line along the interface or by kinking out of the interface into one of the bonded materials. This competition can be assessed by comparing the normalized energy release rate \( G / G^c \), to \( G_0 / G_0^c \) and if

\[
\frac{G}{G^c} > \frac{G_0}{G_0^c}
\]

(7.59)

kinking occurs prior to interface growth. Conversely, if the inequality of (7.59) is reversed, the crack meets the condition for continuing advance along the interface. Here, \( G^c \) and \( G_0^c \) are the critical energy release rate for the conducting path kinking out of the interface and for the conducting path growing along the interface, respectively. For the case when \( \gamma = 0.5 \), the normalized energy release rates as a function of \( \omega \) are
shown in Fig. 7.3. It is seen from Fig. 7.3 that the energy release rate is strongly influenced by material properties. In addition, it is found from Fig. 7.3 that the energy release rate is continuous in all ranges of $\omega$, which is different to the electric field intensity factor.

Next, we consider the problem of kinking of conducting path in the inclined anisotropic dielectric materials as shown in Fig. 7.1. Here, the subscript $p$ denotes the principal axes of orthotropy, and $\theta_p^{(1)}$ and $\theta_p^{(2)}$ are the inclined angles of principal axes of orthotropy in material 1 and material 2, respectively. Some electric properties of an inclined orthotropic dielectric bimaterial, $\lambda^{(1)}$, $\lambda^{(2)}$, $\eta^{(1)}$ and $\eta^{(2)}$ can be expressed as function of $\lambda_p^{(1)}$, $\lambda_p^{(2)}$, $\theta_p^{(1)}$ and $\theta_p^{(2)}$ due to the inclination of the principal axes of orthotropy, thus,

$$\lambda^{(m)} = \frac{\lambda_p^{(m)}}{\cos^2 \theta_p^{(m)} + \left(\lambda_p^{(m)} \sin \theta_p^{(m)}\right)^2},$$

$$\eta^{(m)} = \frac{1 - \left(\lambda_p^{(m)}\right)^2}{\cos^2 \theta_p^{(m)} + \left(\lambda_p^{(m)} \sin \theta_p^{(m)}\right)^2},$$

$$\lambda_p^{(m)} = \sqrt{\frac{e_{11}^{(m)}}{e_{22}^{(m)}}}, \ m=1, 2. \quad (7.60)$$

Fig. 7.4 illustrates the normalized electric field intensity factors as function of $\omega$ when $\lambda_p^{(1)} = 4$ and $\lambda_p^{(2)} = 2$. The normalized energy release rates are plotted as function of $\omega$ for the case when $\lambda_p^{(1)} = 4$ and $\lambda_p^{(2)} = 2$ in Fig. 7.5. It is seen from Figs. 7.4 and 7.5 that the inclination angle of the material principal axes has a strong effect on the electric field intensity factor and energy release rate of kinked conducting path. That mean the growth direction of conducting path is strongly affected by the inclination angle of the material principal axes.

### 7.5 Numerical Verification
Finite element analysis is conducted to numerically calculate the energy release rate of the kinked conducting path in an anisotropic dielectric bimaterial. The electrostatic analysis is carried out with commercial program, ABAQUS. We adopted the eight-node, biquadratic, plane strain, piezoelectric quadrilateral, reduced integration element, CPE8RE. Fig. 7.6 shows the mesh configuration of kinked conducting path with kink angle of $\omega = 30^\circ$.

Rice (1968) interpreted the $J$ integral as the rate at which energy is released as a result of the extension of a crack in a nonlinear elastic body. For the dielectric material, if the crack extends along the $x_1$-axis, the two-dimensional $J$ integral is expressed as

$$J = \int_{\Gamma} (W n_i - t_j u_{j,i} + n_i D_i E_i) d\Gamma, \quad i,j = 1,2,$$  \hspace{1cm} (2.7)

which is same to the case of piezoelectric material introduced in Section 2.4. For the two-dimensional dielectric breakdown problem, we can also employ the $J$ integral (2.7) to examine the energy release rate during growth of conducting path. Under purely electric loading condition, $J$ integral expressed in (2.7) may be reduced as

$$J = \int_{\Gamma} (W n_i + n_i D_i E_i) d\Gamma, \quad i,j = 1,2,$$ \hspace{1cm} (7.61)

Here, $W = -D_i E_i / 2$. For the kinked conducting path, we need to consider the transformation of the coordinate system, because direction of the crack extension was deflected from the interface plane. The linear transformation of coordinate system including translation $T$ and rotation $R$, is illustrated in Fig. 7.7. The coordinates, the electric displacements and the electric fields in the new coordinate system after transformation given by

$$x_j^* = R_j (x_j + T_j),$$ \hspace{1cm} (7.62)

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\[ D_i^* = R_{ij} D_j, \quad (7.63) \]
\[ E_i^* = R_{ij} E_j. \quad (7.64) \]

Here, the matrix \( R \) and vector \( T \) are defined by
\[
R = \begin{bmatrix}
\cos \omega & \sin \omega \\
-\sin \omega & \cos \omega
\end{bmatrix}
\quad \text{and} \quad
T = \begin{bmatrix}
-b \cos \omega \\
-b \sin \omega
\end{bmatrix}.
\]

Therefore, we can rewrite the contour-independent \( J \) integral (7.61) in new coordinate system as
\[
J = \int_{\Gamma^*} (W^* n_i^* + n_i^* D_j^* E_j^*) d\Gamma^*, \quad i,j = 1,2, \quad (7.65)
\]
where \( W^* = -D_j^* E_j^*/2 \). We numerically calculated the \( J \) integral (7.65) using ABAQUS with auxiliary program. The \( J \) integral has the physical meaning of the energy release per unit length of the crack extension along the \( x_i^* \)-axis, hence, we have
\[
J = G. \quad (7.66)
\]

The comparison of the numerical solution obtained from (7.65) and the analytic solution obtained from (7.57) for the case when \( \omega = \pi/6 \), \( \alpha = 0.5 \), \( \eta^{(1)} = 0 \) and \( \varepsilon^{(1)} = 1 \) is plotted in Fig. 7.8. It is seen from Fig. 7.8 that the analytic solution agrees well with the numerical solution.

7.7 Summary

Kinking of a conducting path between two dissimilar anisotropic dielectric materials under purely electric loading is analyzed. Dielectric breakdown commonly occurs in the
dielectric materials by formation of conducting path under a critical electric loading. The problem is formulated using a linear transformation method. Based on the formulation, the complete electric fields in the anisotropic dielectric materials are easily obtained from the solutions of the isotropic dielectric materials. The electric field intensity factors and energy release rates of kinked conducting path are obtained for various inclination angles of material principal axes. It is seen that the inclination angle of material principal axes has an important effect on the growing direction of conducting path. Numerical analysis is conducted to examine the validity of the analytic solution, which is obtained based on the linear transform method, using finite element method. It is seen that the numerical solution is very agree with analytic solution.
CHAPTER 8
CONCLUSIONS

Failure behavior of piezoelectric materials under purely electrical loading is investigated. Under purely electrical loading, piezoelectric material commonly shows two kinds of failure modes: fracture by crack growth and dielectric breakdown by formation of tubular channel or conducting path.

First, the dielectric breakdown of an unpoled piezoelectric ceramic PZT807 with a conductive channel is investigated. Cylindrical bar specimens with a conductive channel are used for tests of the breakdown of the unpoled piezoelectric ceramic under purely electrical loads. Narrow tubular channels emanating from the tip of the initial channel are observed in the specimens after breakdown occurs. The channel is usually initiated at the head of the initial channel and propagates in either a straight or an oblique direction. The radius of the tubular channel that is created at the head of the initial channel is insensitive to various types of channel formation. In order to investigate the effect of the shape of the channel head on the electric field and dielectric breakdown, the initial tubular channel with the shape of either a hemispherical head or a flat head is considered. It is shown that the direction of the channel growth depends upon the shape of the head of the initial channel. The problem of a fine tubular channel emanating from the head of the initial channel is also solved numerically to evaluate the three-dimensional $J$ integral at the onset of breakdown in the specimen. It is shown that the $J$ integral for the channel at the initiation of a new channel can be obtained from the solution of the initial channel. The length and radius of the initial channel affect strongly the $J$ integral. The $J$ integral for the channel is directly related to the energy released per unit length of the channel, as a result of channel growth. The $J$ integral is used as a breakdown parameter that governs the breakdown process. The critical $J$ integral at the onset of breakdown is obtained.
Second, dielectric breakdown in a poled PZT807 ceramic with a conductive channel is investigated. Specimens with the shape of a cylindrical bar are used in tests of dielectric breakdown under purely electrical loading. To investigate the effect of poling directions on breakdown behavior, samples with either a positive or a negative poling direction are used in tests of breakdown. Narrow tubular channels are initiated from the surface of the head of the initial channel during dielectric breakdown, and propagated in a straight direction through the specimen. The radius of the new tubular channel is nearly constant for all of the specimens, regardless of the direction of the applied electric field. The growth of the new tubular channel is accompanied with melt of the material during dielectric breakdown. Almost all specimens broke transversely into two parts beginning from the head of the initial channel. Breakdown voltages were experimentally measured through the high voltage supplier during the test of dielectric breakdown. The length of the initial channel is found to influence the breakdown voltage.

The three-dimensional $J$ integral for a tubular channel is introduced as a criterion of breakdown failure. The breakdown due to the propagation of a channel occurs at a critical $J$ value, which is a measure of breakdown resistance. Piezoelectric analyses are carried out with the finite element program to calculate the electric field and the elastic field. The $J$ integral at the onset of breakdown is calculated numerically for all the samples. For the cases of positive and negative poling directions, the critical $J$ integral at the onset of breakdown is obtained. The breakdown resistance is almost independent of the length of the initial channel, while it slightly decreases as the radius of the initial channel increases. The breakdown resistance is insensitive to the direction of the applied electric field.

Third, the failure behavior of piezoelectric ceramics with a conducting crack that are subject to purely electric loading is investigated. The study lies on the experimental observation made by 박재연 (2003). According to his observation, under electrical loads, specimens fail in one of two modes: fracture accompanied with dielectric
discharging and the formation of tubular channels without fracture. The $J$ integral that is widely used in fracture mechanics is introduced for the dielectric breakdown problem. To solve the problems of the growth of the crack and of the tubular channel from the initial conducting crack, piezoelectric analyses are conducted for evaluating the two and three dimensional $J$ integrals at the onset of fracture and breakdown in piezoelectric ceramics, respectively. The critical $J$ integrals at the onset of fracture or breakdown are obtained. A positive electric field can decrease the breakdown resistance of piezoelectric materials, whereas a negative electric field can increase the breakdown resistance of piezoelectric materials. The poling direction also can affect the breakdown resistance of piezoelectric materials.

Fourth, fracture behavior of a piezoelectric bimaterial with an interfacial electrode subjected to purely electric loading is investigated. Plate shape specimens with interfacial electrodes are used for electrical fracture tests. Cracks emanating from the tip of interfacial electrodes are experimentally observed. Most of the specimens with electrode of length of 5mm are fractured by growth of the crack, whereas, most of the specimens with electrode of length of 7mm are damaged by growth of the tubular channel. In addition, the fracture mode is separated two kinds of modes as well, that is crack kinking out of the interface plane and crack growth along the interface. The mutual integral, which has the conservation property, is applied to obtain electric field intensity factors due to purely electrical loading for interfacial electrodes between dissimilar piezoelectric materials. The critical electric field intensity factors at the onset of crack growth are obtained.

Fifth, kinking of a conducting path between two dissimilar anisotropic dielectric materials under purely electric loading is analyzed. Dielectric breakdown commonly occurs in the dielectric materials by formation of conducting path under a critical electric loading. The problem is formulated using a linear transformation method. Based on the formulation, the complete electric fields in the anisotropic dielectric materials are easily obtained from the solutions of the isotropic dielectric materials. The electric field
intensity factors and energy release rates of kinked conducting path are obtained for various inclination angles of material principal axes. It is seen that the inclination angle of material principal axes has an important effect on the growing direction of conducting path. Numerical analysis is conducted to examine the validity of the analytic solution, which is obtained based on the linear transform method, using finite element method. It is seen that the numerical solution is very agree with analytic solution.
REFERENCES


Zhang, T.-Y., Wang, T., Zhao, M., 2003. Failure behavior and failure criterion of


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\[ \phi = -2 \text{Re} \left[ \sum_{M=1}^{4} B_{aM} f_M(z_M) \right] \]

**Fig. 6.5** Finite element model of a semi-infinite electrode embedded between two dissimilar anisotropic piezoelectric materials

**Fig. 6.6** Critical electric field intensity factors

\[ K_{EC} (\text{kV/} \sqrt{\text{m}}) \]

- \( a \) (mm)

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Fig. 7.5 Normalized energy release rates near the tip of kinked conducting path in an anisotropic dielectric bimaterial as a function of $\gamma$ for various combinations of $\lambda^{p(1)}$, $\theta^{p(1)}$, $\lambda^{p(2)}$ and $\theta^{p(2)}$, when $\gamma = 0.5$. 
Fig. 7.6 Mesh configuration of kinked interface conducting path

\[ \phi = -2 \text{Re} \left[ \sqrt{\frac{z}{2\pi}} K_E \right] \]

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\[ w = \frac{p}{6}, \quad a = 0.5, \quad \eta^{(1)} = 0 \]

- Analytic Solution
- Finite element method

\[ \frac{G}{G_0} \]
Tables

Table 1 Material properties for the PZT807 ceramic

<table>
<thead>
<tr>
<th>Elastic constant (x 10^{10} N/m^2)</th>
<th>Piezoelectric constant (C/m^2)</th>
<th>Dielectric constant (x 10^9 C/Vm)</th>
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<td>$C_{11} = 10.43$</td>
<td>$e_{11} = 11.77$</td>
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<td>$C_{23} = 7.22$</td>
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<td>$C_{33} = 14.26$</td>
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<td>$C_{55} = 3.19$</td>
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Table 2 Material properties for the PZT DE-DL ceramic

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Table 3 Material properties for the PZT C-201 ceramic

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<th>Piezoelectric constant (N/Vm)</th>
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<td>$C_{33} = 93.76$</td>
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<td>$C_{35} = 23.91$</td>
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Table 4 Material properties for the PZT C-3 ceramic

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Table 5 Numerical verification

<table>
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<th>Applied electric field intensity factor $K_{E}^{appl}$ (GV/$\sqrt{\text{m}}$)</th>
<th>Obtained electric field intensity factor $K_{E}$ (GV/$\sqrt{\text{m}}$)</th>
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<tr>
<td>1.00</td>
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<tr>
<td>2.00</td>
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감사의 글

우선 많이 부족했던 저에게 6년간의 유학생생활을 박사학위 취득으로 완만하게 마무리 짓도록 세심한 지도와 배려를 아끼지 않았으신 범현규교수님께 진심으로 고개 숙여 깊은 감사의 인사를 올립니다. 6년 전 인생의 역전을 꿈꾸며 한국땅을 처음 밟았을 때의 제 모습이 떠오릅니다. 나름대로의 기대감에 설레기도 했지만 내심으로는 이국문화에 대한 생소함에 많이 두려웠기도 했습니다. 하지만 교수님의 세심한 배려덕분에 이러한 두려움을 잊고 학업에 정진할 수 있었습니다. 다시 한번 고개 숙여 감사 드립니다. 바쁜 와중에도 저의 학위논문심사를 빠르게 맡기어 주신 김창부교수님, 이우식교수님, 전남대학교 양영수교수님, 김정엽교수님께 진심으로 감사 드립니다.

인하대학교 전자재료역학연구실에서 6년간 통합과정으로 있으면서 참 많은 선후배님들의 도움이 있었습니다. 가까이에서 저의 부족한 부분들을 강세주고 정서적으로 안정을 되찾고 하루빨리 학업에 정진할 수 있도록 이끌어주신 유환형, 1년 동안 합숙하면서 한국문화와 한국인에 대해 많이 가르쳐주신 창호형, 당시 연구실 유일한 동갑내기 친구로 1년간 합숙하면서 진심에 못지않게 어려모로 많이 챙겨주신 기현이, 같은 중국인으로 학부도 중국에서 함께 다녔고 대학원도 한국에서 함께 다니면서 많은 도움을 준 대홍이, 대학원에 함께 진학하여 나름대로의 끈끈한 동기애를 보여주신 대홍이, 창호형, 보현이, 막내 현석이, 이 모든 분들에게 진심으로 고개 숙여 감사 드립니다. 그리고 긴 시간을 함께한 시스템 고체역학연구실에 반강, 록성, 동학연구실에 정우형, 세희누나, 보연, 정기, 격려와 도움에 감사 드립니다. 그리고 만날 때마다 격려와 조언을 아끼지 않았으신 정경문박사님과 지숙이누나에게도 감사 드립니다.

제가 인하대학교 대학원에 입학할 수 있게 알게 모르게 많이 힘써주시고 가까이에서 지켜보면서 진정생명 챙겨주시고 힘들 때 따뜻한 격려의 말로 다독여주셨던 이봉형, 창호형께 진심으로 감사 드립니다. 한국에 와서 친하게 되었
지만 모든 면에서 서로가 너무나 잘 통했고 또 그래서 너무나 편하고 좋았던, 항상 만나면 서로의 꿈과 희망을 함께 나누며 한국에서의 성공을 기약했던 절친,춘호-영숙부부, 오철-련화부부에게도 진심으로 감사 드립니다. 그리고 인하대에서 함께 공부하면서 많은 격려와 도움을 주신 철준형, 은철형과 연봉하박사님께도 감사 드립니다.

2010년 여름, 좋은 사람을 만나 행복한 가정을 이루게 되었습니다. 힘들 때나 기쁨 때나 항상 저의 곁을 지켜주고, 마지막까지 잘 마무리 할 수 있도록 큰 힘이 되어준 저의 평생의 반려자 금화와 이 기쁨을 함께 나누고자 합니다. 그러고 사위가 아닌 친아들에 대한 사랑으로 보살펴주시는 장인어른과 장모님께도 진심으로 감사 드립니다. 그리고 지금까지 많은 도움과 성원을 보내주신 외할아버지, 외할머니, 큰고모부, 큰고모, 삼촌들, 숙모들, 아모들, 이모부들, 고모부들, 동생들, 일일이 열거할 수 없지만 이 모든 분들께 진심으로 감사 드립니다.

오늘이 있기까지 무엇보다도 부모님들의 현신적인 사랑이 있었습니다. 넉넉하지 않은 생활이었음에도 불구하고 항상 저에 대한 투자를 아끼지 않았습니다. 저를 위해 충고 어두운 면 시베리아 땅까지 밟으셨습니다. 자신보다는 저를 위한 삶을 살아오셨던 부모님들의 현신적인 사랑과 노고에 감사 드리며 이 논문을 당신들에게 바칩니다.