Self-imaging phenomena in multi-mode photonic crystal line-defect waveguides: application to wavelength de-multiplexing

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Abstract: We show that the self-imaging principle still holds true in multi-mode photonic crystal (PhC) line-defect waveguides just as it does in conventional multi-mode waveguides. To observe the images reproduced by this self-imaging phenomenon, the finite-difference time-domain computation is performed on a multi-mode PhC line-defect waveguide that supports five guided modes. From the computed result, the reproduced images are identified and their positions along the propagation axis are theoretically described by self-imaging conditions which are derived from guided mode propagation analysis. We report a good agreement between the computational simulation and the theoretical description. As a possible application of our work, a photonic crystal 1-to-2 wavelength de-multiplexer is designed and its performance is numerically verified. This approach can be extended to novel designs of PhC devices.

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OCIS Codes: (130.3120) Integrated Optic devices; (130.1750) Components

References and links
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1. Introduction

Since photonic band gaps (PBGs) were known to exist in periodically modulated dielectric materials [1,2], which are the so-called photonic crystals (PhCs), a large number of studies have been devoted to revealing their unique properties [3]. These artificial sub-micro structures provide flexible controls of light by breaking periodicity and by introducing defects. Recently, well-established fabrication technologies and advanced numerical methods have enabled the sophisticated engineering of PhC dispersion characteristics [4-6].

In devices based on PBG, well-known lightwave behaviors, such as directional coupling and resonance effect in cavities, have been exploited as well as in conventional optical devices based on total internal reflection (TIR), and their characteristics have been thoroughly investigated in many papers [7-9]. Compared with those in conventional TIR devices, the well-known behaviors of lightwave in PBG devices are observed in much smaller operating regions, due to the PhC’s ability to interact with light on a wavelength scale. For instance, the directional coupling between adjacent line-defect waveguides occurs within a few wavelengths [10,11]. This strong interaction with light in PBG devices is expected to usher in an era of ultra-compact lightwave circuits that operate with much improved functionalities.

While such lightwave behaviors have been successfully adopted into PBG devices, they are assumed to take place in structures that ensure single-mode operation for the robust performance of devices. Hence, few studies in the available literatures focus primarily on discussing PhC line-defect waveguides (PCWs) that support multi-mode operation. Furthermore, researches on self-imaging phenomena that might be observed in multi-mode PCWs, to our knowledge, have not been reported so far. In this paper, we observe the propagation of lightwaves in multi-mode PCWs to determine if self-imaging phenomena occur and, if so, to confirm that they can be utilized to control lightwaves in an ultra-compact structure.

To demonstrate self-imaging phenomena in multi-mode PCWs, a numerical computation is performed with the finite-difference time-domain (FDTD) method [12]. We pattern a multi-mode PCW on the computational domain by removing five consecutive rows of dielectric rods in otherwise perfect crystals and add to its entrance a one-line defect PCW, through which an input field is launched to excite the guided modes in the multi-mode PCW. We observe that modal interferences by these excited modes cause the input field to be reproduced along the multi-mode PCW. By employing self-imaging conditions derived from the guided mode propagation analysis, the positions of reproduced images in the propagation direction are described. As an application of our work, a 1-to-2 wavelength de-multiplexer is designed and numerically demonstrated for the validation of self-imaging in multi-mode PCWs.

2. Self-imaging phenomena in multi-mode PCWs

2.1 Brief review of self-imaging and numerical experiment
In this sub-section, the self-imaging principle in conventional dielectric waveguides is briefly reviewed, and we investigate the occurrence of self-imaging phenomena in multi-mode PCWs by means of FDTD calculations.

We begin with defining the concept of self-imaging: Self-imaging is a property of multi-mode waveguides by which an input field profile is reproduced in single or multiple images at periodic intervals along the propagation axis [13]. From the above definition, an input image can be reproduced in single or multiple images but, for the sake of simplicity, reproduction of only a single image is considered in this paper. As shown in Fig. 1, if an input field $\Psi(0,y)$ is introduced into a multi-mode waveguide at $x=0$ with an asymmetric displacement $d$ from the plane $y=0$, two kinds of images are reproduced at $x=L_m$ and $x=L_d$, depending on self-imaging conditions: one is a replica of the input field mirrored with respect to the plane $y=0$ at $x=L_m$ and the other is a direct replica of the input image at $x=L_d$.

For a conventional waveguide, like the one presented above, self-imaging can be accepted without doubt or it may well be taken for granted. However, for a multi-mode PCW, a natural question arises: if an input field is injected into a multi-mode PCW, can self-imaging phenomena still be observed? A numerical simulation can give a satisfactory answer to this question. Our setup for the simulation is similar to that of Fig. 1 except that the conventional multi-mode waveguide is replaced with a PhC equivalent, as shown in Fig. 2. For simplicity, we assume a 2-dimensional photonic crystal structure consisting of a square lattice of dielectric rods in air. In this structure, five consecutive rows are removed in otherwise perfect crystals to form a multi-mode PCW. The refractive index of the dielectric rods is set to be 3.4 and their radius is 0.18$a$, where $a$ is the lattice constant of the crystal. In this crystal, the band gap opens for the frequency range of 0.303-0.445($a$/\lambda) for the E-polarization (electric field parallel to the rods), where $\lambda$ is the wavelength in free space.
As an access waveguide, a one-line-defect PCW is introduced into the multi-mode PCW as shown in Fig. 2. This access PCW supports single-mode, and ensures that a well-confined input field is injected into the multi-mode PCW for practical analysis.

Before launching an input field into the multi-mode PCW, we should know the property of guided modes in the PCWs because self-imaging is attributed to the multi-mode interference, which strongly depends on the number of modes, propagation constants, and modal field patterns. To confirm the number of guided modes supported by the access PCW and by the multi-mode PCW, the dispersion curves for the two PCWs are presented in Fig. 3. The dispersion curves are calculated by the plane wave expansion (PWE) method [14] and the computational super-cells used for the calculation are depicted as the insets in Fig. 3(a) and (b).

![Dispersion Curves](image1.png)

Fig. 3. (a) The dispersion curve for the access PCW and the computational super-cell (inset). The access PCW ensures single-mode operation from $0.312(\alpha/\lambda)$ to the top of band gap. (b) The dispersion curve for the multi-mode PCW and the computational super-cell (inset). The multi-mode PCW supports 4 guided modes at $0.37(\alpha/\lambda)$ and 5 guided modes at $0.43(\alpha/\lambda)$.

While the frequency range of single-mode operation for the access PCW extends from $0.312(\alpha/\lambda)$ to the top of the band gap, as shown in Fig. 3(a), the multi-mode PCW supports from three to five guided modes for the same frequency range (Fig. 3(b)). We choose an operating frequency of $0.37(\alpha/\lambda)$ within the frequency range where the multi-mode PCW supports more than three guided modes (higher than $0.35(\alpha/\lambda)$), as presented in Fig. 3(b).

![Modal Patterns](image2.png)

Fig. 4. Modal patterns of electric field $z$-component for each mode at the operation frequency $0.37(\alpha/\lambda)$ presented in Fig. 3. (a) Input image for the access PCW, (b) the 0th mode, (c) the 1st mode, (d) the 2nd mode, and (e) the 3rd mode at $0.37(\alpha/\lambda)$. 
To identify field patterns of guided modes in the access PCW and the multi-mode PCW, the modal field distributions are calculated at the operating frequency by the PWE method. Fig. 4 shows the z-component of the electric field at each calculation point, which are marked on dispersion curves in Fig. 3. The modal numbers are assigned to dispersion curves in Fig. 3(b) in accordance with the modal patterns presented in Fig. 4. The modal field patterns have their own symmetry, even or odd, with respect to the propagation axis, hence they can be selectively excited depending on the input position. However, in our case, since the access PCW is introduced into the multi-mode PCW with an asymmetric position of $y = 2/a$, as shown in Fig. 2, all the modes of the multi-mode PCW at the frequency of $0.37(a/\lambda)$ are excited by the input field (Fig. 4(a)), which is confirmed by the overlap integral. Therefore, they all contribute to self-imaging.

The configuration of Fig. 2 is directly transferred to the FDTD computational domain for our numerical experiment. The domain is surrounded by perfectly matched layers to absorb the outgoing waves. A continuous wave at the frequency of $0.37(a/\lambda)$ is launched into the access PCW in Fig. 2, and the distributions of steady-state electric field and time-averaged Poynting vector are obtained after sufficient time steps, which are shown in Fig 5. In these Figures, especially in the time-averaged Poynting vector distribution in Fig. 4 (b), it is likely that two kinds of images are reproduced by self-imaging; one is expected to be a mirrored replica at $x=21a$ and the other to be a direct replica at $x=31a$.

Fig. 5. (a) Steady-state electric field distribution at $0.37(a/\lambda)$. (b) Time-averaged Poynting vector distribution at $0.37(a/\lambda)$.

In the following sub-section, the above observations are theoretically supported by self-imaging conditions. For this purpose, the general self-imaging conditions for the mirrored and direct replicas are derived, and are used to describe the positions of the replicas along the propagation axis.

### 2.2 Analysis of multi-mode PCWs using the self-imaging principle

Here, we present some mathematical derivations in an attempt to describe what we have observed in the previous section. To derive mathematical conditions for self-imaging at $x=L_m$ and at $x=L_d$ in Fig 1, it is assumed that the input image $\Psi(0,y)$ is well confined to the guiding region not to excite the radiation modes so that the total field $\Psi(x,y)$ in the multi-mode waveguide at a distance $x$ can be expressed as a sum of all guided modes:

$$\Psi(x, y) = \sum_{n=0}^{p-1} c_n \varphi_n(y) e^{-j\beta_n x}, \quad (1)$$
where $c_n$ is the field excitation coefficient, $\phi_n(y)e^{j\beta_n}y$ is the modal field distribution with the propagation constant $\beta_n$, $p$ is the number of modes, and the subscript $n$ denotes the order of the mode ($n=0, 1, 2, \ldots p-1$).

For the mirrored image to be reproduced at $x=L_m$, the total field after propagating the length $L_m$, $\Psi(L_m, y)$, should become

$$
\Psi(L_m, y) = \sum_{n=0}^{p-1} c_n \phi_n(y)e^{-j\beta_n L_m} = c_0 \phi_0(y)e^{-j\beta_0 L_m} + c_2 \phi_2(y)e^{-j\beta_2 L_m} + c_4 \phi_4(y)e^{-j\beta_4 L_m} + \cdots,
$$

(2a)

$$
= c_0 \phi_0(y)e^{-j\beta_m} + c_2 \phi_2(y)e^{-j\beta_2 L_m} + c_4 \phi_4(y)e^{-j\beta_4 L_m} + \cdots
$$

$$
= \Psi(0, -y)e^{-j\Delta_m}
$$

where $\Psi(0, -y)e^{j\alpha}$ is the mirrored input image with an arbitrary constant phase term $\Delta_m$ and it is again decomposed for a term-by-term comparison with $\Psi(L_m, y)$:

$$
\Psi(0, -y)e^{-j\Delta_m} = \sum_{n=0}^{p-1} c_n \phi_n(-y)e^{-j\alpha_n} = c_0 \phi_0(y)e^{-j\alpha_m} + c_2 \phi_2(y)e^{-j\alpha_2} + c_4 \phi_4(y)e^{-j\alpha_4} + \cdots.
$$

(2b)

We use the structural symmetry of modal field distributions with respect to the plane $y=0$ in Eq. (2b):

$$
\phi_n(-y) = \left\{ \begin{array}{ll}
\phi_n(y) & \text{for } n \text{ even} \\
-\phi_n(y) & \text{for } n \text{ odd} 
\end{array} \right.
$$

(3)

By inspecting the phase relations between terms in Eq. (2a) and those in Eq. (2b), and by comparing them one by one, the condition for the mirrored image at $x=L_m$ is expressed as

$$
\beta_n L_m = \left\{ \begin{array}{ll}
2k_n \pi + \Delta_m & \text{for } n \text{ even} \\
(2k_n -1) \pi + \Delta_m & \text{for } n \text{ odd}
\end{array} \right. \text{ with } k_n = 1, 2, 3 \ldots
$$

(4)

As with the condition for the mirrored image at $x=L_m$, the condition for the direct replica at $x=L_d$ is derived similarly. It should be expressed in the following form:

$$
\beta_n L_d = 2k_n \pi + \Delta_d \quad \text{with } k_n = 1, 2, 3 \ldots,
$$

(5)

where $\Delta_d$ is an arbitrary constant phase term for the direct replica.

We use Eq. (4) and (5) to describe the positions where the input image is reproduced. Given the values of propagation constants at an operation frequency, Eq. (4) and (5) produce a set of simultaneous equations, individually, in which each set has the same number of equations as in the propagation constants. Hence, to determine the positions of the reproduced images, each of the two equation sets should be simultaneously solved. For the detailed procedure of applying Eq. (4) and (5) to our case, the values of propagation constants at the operating frequency of 0.37(\alpha/\lambda) are taken out from the dispersion curves in Fig. 3(b) and are then plugged into Eq. (4) and (5). In general, exact solutions to Eq. (4) and (5) do not exist due to the periodic nature of sinusoidal functions, but it is possible to determine the nearest positive integer sets, $\{k_n\}$, ($n=0, 1, 2, 3, \text{ in our case}$) that are simultaneously fit into the equation sets with acceptable errors. By assuming $\Delta_m=0$ and $\Delta_d=0$, for the sake of simplicity,
we first try to find such integer sets for Eq. (4) and (5), respectively, so that they can minimize the discrepancy in values of \( L_m \) and \( L_d \). Then, \( \Delta_m \) and \( \Delta_d \) are determined by means of the least squares method, which further decreases the deviation from the mean values of \( L_m \) and \( L_d \). The values used to calculate \( L_m \) and \( L_d \) are summarized in Tables 1 and 2, respectively.

### Table 1. Parameters Used to Calculate \( L_m \) at 0.37(\( a/\lambda \))

<table>
<thead>
<tr>
<th>( n )</th>
<th>( k_n )</th>
<th>( \beta_n (2\pi/a) )</th>
<th>( L_m = \frac{(2k_n \pi + \Delta_m)}{\beta_n} ) for ( n ) even</th>
<th>( L_m = \frac{(2k_n \pi + \Delta_m)}{\beta_n} ) for ( n ) odd</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>13</td>
<td>0.3600</td>
<td>30.769231a</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>13</td>
<td>0.3267</td>
<td>31.381804a</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>0.2634</td>
<td>31.294452a</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>0.1334</td>
<td>31.680954a</td>
<td></td>
</tr>
</tbody>
</table>

Mean value of \( L_m \) = 31.28161025a

*We first find \( \{k_n\} \) and \( L_m \) assuming \( \Delta_m=0 \), and then determine \( \Delta_m=0.266483\pi \) by least squares method.

### Table 2. Parameters Used to Calculate \( L_d \) at 0.37(\( a/\lambda \))

<table>
<thead>
<tr>
<th>( n )</th>
<th>( k_n )</th>
<th>( \beta_n (2\pi/a) )</th>
<th>( L_d = \frac{(2k_n \pi + \Delta_d)}{\beta_n} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7</td>
<td>0.3600</td>
<td>20.357672a</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>0.3267</td>
<td>20.809157a</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0.2634</td>
<td>19.895764a</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.1334</td>
<td>19.653858a</td>
</tr>
</tbody>
</table>

Mean value of \( L_d \) = 20.17911275a

*We find \( \{k_n\} \) and \( L_d \) assuming \( \Delta_d=0 \), and then determine \( \Delta_d=0.432034\pi \) by least squares method.

Through the comparison of the theoretical description presented above with the numerical simulation in Fig. 5, it is shown that the self-imaging principle is still valid in multi-mode PWGs.

### 3. Design of wavelength de-multiplexer based on self-imaging

In this section, a PhC de-multiplexer is designed by using the self-imaging conditions derived in the preceding section. Our goal is to de-multiplex two wavelengths so that a 1-to-2 structure is required for routing each wavelength to a corresponding output. As shown in Fig. 6, we simply add two output PCWs to the structure presented in Fig. 2 and the length of the multi-mode PCW is set to be 31\( a \), since the direct replica at 0.37(\( a/\lambda \)) is imaged at around 31\( a \).
This wavelength at 0.37($a/\lambda$) is intended to be routed to output port A. The remaining job is to choose another wavelength to be routed to output port B. Scanning computation of several frequency points on dispersion curves in Fig. 3(b) makes it possible to determine another appropriate operating point of 0.43($a/\lambda$) at which a mirrored replica is imaged at almost the same position as the direct replica is imaged. Also, this design is directly applicable to a 1.5/1.3µm de-multiplexer by setting the lattice constant, $a$, as 570nm. Unlike the previous situation at 0.37($a/\lambda$), here there are five guided modes at 0.43($a/\lambda$) to be considered—one more guided mode than at 0.37($a/\lambda$), as shown in Fig. 3. However, the 4th mode is not excited, since it has odd symmetry with respect to the input field. Hence, four out of five modes are considered in the calculation of Eq. (4). Fig. 7 shows the steady-state electric field distributions as obtained by FDTD calculations after two continuous waves at 0.37($a/\lambda$) and 0.43($a/\lambda$) are launched into the access PCW.

To measure the transmission characteristic of the PhC de-multiplexer, we calculate the time-averaged Poyning vectors at the output port A and B, for the final ten periods after the FDTD simulated electric field distributions reached steady-state. According to the previous paper [15], the losses at 90° bends in output PCWs are below 5% at the two wavelengths we choose as operating points. The calculated output power is normalized to the total input power, as shown in Table 3.
Table 3. Output Power Normalized to Total Input Power

<table>
<thead>
<tr>
<th>Frequency (a/λ)</th>
<th>Output A</th>
<th>Output B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.37</td>
<td>95.2%</td>
<td>2.3%</td>
</tr>
<tr>
<td>0.43</td>
<td>4.2%</td>
<td>92.4%</td>
</tr>
</tbody>
</table>

4. Conclusion

We show that the self-imaging principle still holds true in multi-mode photonic crystal (PhC) waveguides just as it does in conventional multi-mode waveguides. Theoretical predictions are presented based on self-imaging conditions for both the direct and the mirrored replica, and results of the FDTD simulation are compared with them. A good agreement between theory and numerical experiment is reported. From this result, we designed a PhC wavelength de-multiplexer based on the self-imaging phenomena that occur in multi-mode PCWs so that multi-mode PCWs can be viewed as a new platform to design novel ultra-compact PBG devices that utilize self-imaging phenomena. However, there is still much room for further research on self-imaging phenomena in multi-mode PCWs. Researches on multiple images, for instance, would make it possible to design various PhC devices, such as 1-to-N power-splitters. Also, this work is solely based on the 2-dimensional (2D) properties of photonic crystal waveguides of the square lattice of dielectric pillars in air. For practical applications in integrated optics, however, it is important to investigate the system of air holes in a semiconductor slab; we showed the preliminary results of the PhC waveguides with a triangular lattice of air holes in the dielectrics in Ref. 16. Moreover, based on the results, a study on multi-mode interference of the 2D PhC slab waveguides (full 3-dimensional system) to be considered the light line is in progress according to the proposed procedure.

Finally, it is also worth noting that, although self-imaging phenomena in multi-mode PCWs are clearly observed in the numerical simulation and can be well predicted by self-imaging conditions, they are different from those in conventional waveguides. Firstly, because the lateral guiding mechanism of PCWs depends on PBG guidance, not on TIR, useful approximations, such as weakly guiding approximation, cannot be applied to them, leading to difficulties in determining generalized design formulations. Secondly, since PCWs have discrete structure in size in terms of lattice periodicity, this can limit the degree of freedom in structural design; as a consequence, the length of devices cannot be designed as determined by self-imaging conditions, which can cause the device to suffer from large additional losses.

Acknowledgments

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