Performance and Distance Spectrum of Space-Time Codes in Fast Rayleigh Fading Channels

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Abstract—In this paper, we analyze the performance of space-time codes and propose a distance spectrum computation method in fast Rayleigh fading channels. We first derive a new FER upper bound using the union bound and the PEP upper bound in the fast fading environment. The derived FER upper bound is very accurate, requires only the distance spectrum of the space-time code, and takes a closed-form expression. Then we propose a distance spectrum computation method of space-time codes in fast fading channels, which exploits the symmetric property of the error state diagram in space-time trellis coded MPSK modulation to reduce the computation complexity. Numerical results illustrate that the derived FER bound is tight enough to estimate the performance of space-time codes in fast fading channels with sufficient accuracy.

Index Terms—Space-time codes, frame error rate, fast fading, distance spectrum.

I. INTRODUCTION

RECENTLY, space-time codes have been introduced as an effective means to achieve high data rates in wireless communication environments [1]. This technique integrates channel coding, modulation, and multiple transmit antennas at the base station, with optional receive diversity incorporated at the mobile station. The pairwise error probability (PEP) and the distance spectrum are two key elements in the design and analysis of space-time codes. Tarokh et al. [1] derived a simple but loose upper bound for the PEP using the Chernoff bound, and presented design guidelines for space-time trellis codes based on the PEP bound for slow and fast fading channels. Fitz et al. [2] derived the exact PEP for slow fading channels and proposed an upper bound for the PEP which is tighter than the Tarokh’s bound. Recently Byun and Lee [3] presented a tight PEP upper bound which is even tighter than the Fitz bound.

For a better design and analysis, the distance spectrum of space-time codes should be considered. In the case of slow fading channels, Aktas and Fitz [4], [5] presented a performance analysis technique based on the distance spectrum of space-time codes and Byun, Park, and Lee [6] recently presented a tight frame error rate (FER) upper bound using the distance spectrum. In the case of fast fading channels, Uysal and Georgiades [7], [8] derived the exact PEP in residue integration form and Simon [9] derived the exact PEP in numerical integration form. They also analyzed the bit error probability performances considering some dominant error events. In the analyses, only the case when the all-zero codeword is transmitted is considered, but this can cause seriously misleading results as space-time codes are not geometrically uniform in general. Moreover, the evaluation of the FER estimates requires the knowledge of all the Euclidean distances of each error event, which would be a considerable burden in memory space and computation. As the FER estimates do not take closed-form expressions, numerical or residue integrations are needed for the computation.

In this paper, we analyze the performance of space-time codes in fast Rayleigh fading channels by deriving a new FER upper bound and devising a distance spectrum computation method. We use a simple union bound and the PEP upper bound to derive a new FER upper bound which is a function of the distance spectrum of the space-time code. The newly derived bound is very accurate but requires only the product distances of error events and takes a closed-form expression. Then we apply the concept of distance spectrum developed for use in slow fading channels [4], [5] to fast fading channels, devising a fast method of distance spectrum computation. Using this process we consider the cases of all possible transmitted codewords but exploit the symmetry of the error state diagram to reduce the complexity of the distance spectrum computation.

The paper is organized in the following manner: We first describe the system model of the communication system employing space-time codes in Section II and derive a new FER upper bound in Section III. Then we describe the distance spectrum of space-time codes in fast fading channels in Section IV and propose its computation algorithm in Section V. Finally, in Section VI, we compare the performance of the new FER bound with simulation results.

II. SYSTEM MODEL

We consider a baseband communication system with \( n_T \) transmit antennas and \( n_R \) receive antennas. The transmitted data are encoded by a space-time encoder. The coded data sequence is applied to a parallel-to-serial (S/P) converter that produces \( n_T \) parallel data sequences. At each time instant \( t \), the \( n_T \) parallel outputs \( c_{\ell T}, c_{\ell T}^2, \cdots, c_{\ell T}^{n_T} \) of the same duration are simultaneously transmitted by the \( n_T \) transmit antennas,
with the symbol $c_i^t$ transmitted by antenna $i$, $1 \leq i \leq n_T$.
We assume that the frame length is $L$, and the elements of the
signal constellation are constructed such that the average
energy of the constellation becomes 1. We define a space-time
codeword matrix $c$ of size $n_T \times L$, obtained by arranging the
transmitted sequence in an array, as
\[
c = \begin{bmatrix}
c_1^1 & c_1^2 & \cdots & c_1^L \\
c_2^1 & c_2^2 & \cdots & c_2^L \\
\vdots & \vdots & \ddots & \vdots \\
c_{n_T}^1 & c_{n_T}^2 & \cdots & c_{n_T}^L 
\end{bmatrix},
\]
where the $i$-th row $c_i^t = [c_i^1, c_i^2, \cdots, c_i^L]$ is the data
sequence transmitted from the $i$-th transmit antenna, and the $t$-th column
$\mathbf{c}_t = [c_1^t, c_2^t, \cdots, c_{n_T}^t]^T$ is the space-time symbol at time $t$.

At the receiver, the signal received at each of the $n_R$ receive
antennas is a noisy superposition of $n_T$ transmitted signals
degraded by channel fading. Let $r \equiv [r_1, r_2, \cdots, r_L]$ denote
the received signal sequence with $r_t = [r_1^t, r_2^t, \cdots, r_{n_R}^t]^T, 
\quad t = 1, 2, \cdots, L$. The received signal at time $t$ and receive
antenna $j$, $j = 1, 2, \cdots, n_R$, is given by
\[
r_t^j = \sqrt{E_s} \sum_{i=1}^{n_T} a_{i,j} c_i^t + \eta_t^j,
\]
where $E_s$ denotes the energy per symbol and $\eta_t^j$ the noise
component of the receive antenna $j$ at time $t$, which is an independent sample of the zero-mean complex Gaussian random variable with variance $\frac{E_s}{n_T}$ per dimension. The coefficient $a_{i,j}$ is the fading attenuation for the path from transmit antenna $i$ to receive antenna $j$. The received signals at time $t$ are related to the transmitted space-time signal by
\[
r_t = \sqrt{E_s} \mathbf{a}_t c_t + \eta_t,
\]
where $\mathbf{a}_t$, the fading coefficient matrix at time $t$, is given by
\[
\mathbf{a}_t = \begin{bmatrix}
a_{1,1} & a_{1,2} & \cdots & a_{1,n_T-1} \\
a_{1,2} & a_{2,2} & \cdots & a_{2,n_T-2} \\
\vdots & \vdots & \ddots & \vdots \\
a_{1,n_R} & a_{2,n_R} & \cdots & a_{n_T,n_R} 
\end{bmatrix},
\]
and $\eta_t$, the noise component vector, is given by
\[
\eta_t = [\eta_t^1, \eta_t^2, \cdots, \eta_t^{n_R}]^T.
\]

In this paper, we assume that the signals received at different
antennas experience independent fading, which means that the fading
coefficients $a_{i,j}$ are independent complex Gaussian random
variables with a zero mean and the variance of $1/2$ per dimension. In addition, we consider that the path coefficients can be modeled as fast Rayleigh fading. For fast fading, it is assumed that the fading coefficients are constant within each symbol period but vary symbol to symbol.

Now we introduce the generator matrix notation [10] to define a space-time trellis coded modulation (TCM). We consider a space-time trellis coded M-PSK modulation with $n_T$ transmit antennas. At time $t$, $R = \log_2 M$ binary inputs $a_1^t, a_2^t, \cdots, a_R^t$, with $a_k^t \in \{0, 1\}$, are fed into a trellis encoder comprised of $R$ feedforward shift registers. The $k$-th input $a_k^t$, $k = 1, 2, \cdots, R$, is first passed to the $k$-th shift register. Then, it is subsequently delayed and multiplied by the encoder coefficient set, given by the generator matrix
\[
G_k = \begin{bmatrix}
g_{k,1} & g_{k,2} & \cdots & g_{k,1} \\
g_{k,2} & g_{k,2} & \cdots & g_{k,2} \\
\vdots & \vdots & \ddots & \vdots \\
g_{k,n_T} & g_{k,n_T} & \cdots & g_{k,n_T} 
\end{bmatrix}.
\]
where $g_{k,i} \in \{0, 1, \cdots, M - 1\}$ is an index of an M-PSK
constellation element, and $v_k$ is the memory order of the $k$-
th shift register. The multiplier outputs are added modulo $M$, giving the encoder output
\[
f_t^j = \sum_{k=0}^{n_T} g_{k,i} a_{k-i} \mod M, \quad i = 1, 2, \cdots, n_T.
\]
The encoder outputs are mapped into signals from the M-PSK
codestory by
\[
c_i^t = \exp \left( \frac{2\pi i c_i}{M} \right), \quad i = 1, 2, \cdots, n_T.
\]
Modulated signals form the space-time symbol $c_i^t = [c_1^t, c_2^t, \cdots, c_{n_T}^t]^T$, the $t$-th column of $c$. These signals are simultaneously transmitted by the $n_T$ transmit antennas.

The space-time trellis coded M-PSK can achieve a bandwidth
width efficiency of $R$ bits/Hz. The total memory order of the encoder, $\nu$, is given by
\[
\nu = \sum_{k=1}^{R} \nu_k,
\]
where the value of $\nu_k$, $k = 1, 2, \cdots, R$, denotes the memory
order for the $k$-th input bit. The number of states for the trellis encoder is $2^R$. In summary, the generator matrices of a space-time TCM, with $R$ input bits and one output space-time symbol per state transition and memory $\nu$, are a set of $R$ matrices $G_1, G_2, \cdots, G_R$ of size $n_T \times (\nu_k + 1)$ each.

\section{Performance Analysis of Space-Time Codes}
For the performance analysis of space-time codes in fast
Rayleigh fading channels we first examine the PEP of space-
time codes and then derive the FER upper bound using this
PEP and the union bound.

\subsection{Pairwise Error Probability}
The PEP is the probability that the decoder selects the sequence $\hat{c} = (\hat{c}_1, \hat{c}_2, \cdots, \hat{c}_L)$ as the estimate of the transmitted sequence $c = (c_1, c_2, \cdots, c_L)$. This occurs if
\[
\sum_{t=1}^{L} \sum_{j=1}^{n_T} \left| r_t^j - \sqrt{E_s} \mathbf{a}_t c_t^j \right|^2 \geq \sum_{t=1}^{L} \sum_{j=1}^{n_T} \left| \mathbf{a}_t c_t^j - \mathbf{a}_t \hat{c}_t^j \right|^2,
\]
which is equivalent to
\[
\sum_{t=1}^{L} \sum_{j=1}^{n_R} \frac{2}{E_s} \text{Re} \left\{ \mathbf{a}_t^j \mathbf{a}_t \hat{c}_t^j \right\} \left( \hat{c}_t^j - c_t^j \right) \right|^2.
\]
If an ideal channel state information (CSI) is available at the receiver, for a given fading channel coefficient $\alpha_t$, the term on the right-hand side of (11) becomes a modified squared Euclidean distance between the two space-time codeword matrices $c$ and $\hat{c}$, denoted by $d^2(c, \hat{c})$, and the term on the left-hand side becomes a zero-mean Gaussian random variable with variance $\frac{1}{2\sigma^2}d^2(c, \hat{c})$, where $\sigma^2$ denotes SNR per symbol. Therefore, the PEP takes the expression [11]

$$P_e(c, \hat{c}) = E\left[Q\left(\sqrt{\frac{\gamma_s}{2}}d^2(c, \hat{c})\right)\right]$$

(12)

for the tail probability of the Gaussian probability density function $Q(y) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{y} e^{-\frac{x^2}{2}} dx$. Here, the expectation is taken over the fading channel coefficients.

In fast fading channels, the distance $d^2(c, \hat{c})$ can be written as [1], [12]

$$d^2(c, \hat{c}) = \sum_{k=1}^{\delta_H} \sum_{j=1}^{n_R} D_k \left| \beta_k^j \right|^2 ,$$

(13)

where $\delta_H$, called the symbol-wise Hamming distance, denotes the number of time instances $t = 1, 2, \cdots, L$, that the space-time symbols $c_t$ and $\hat{c}_t$ differ,

$$D_k \equiv \| c_{t_k} - \hat{c}_{t_k} \|^2 = \sum_{i=1}^{n_T} | c_{t_k}^i - \hat{c}_{t_k}^i |^2$$

(14)

denotes the squared Euclidean distance between the two space-time symbols $c_{t_k}$ and $\hat{c}_{t_k}$. $t_1, t_2, \ldots, t_{\delta_H}$ denote the time instances at which the space-time symbols $c_t$ and $\hat{c}_t$ differ. Also, $\beta_k^j$ is in (13) are independent complex Gaussian random variables with a zero mean and the variance of $\frac{1}{2}$ per dimension. The right-hand side of (13) has $\delta_Hn_R$ independent random variables, so the diversity gain of $\delta_Hn_R$ is attained.

In the case of Rayleigh fading, the PEP in (12) has been evaluated in [3], [9] and expressed as

$$P_p(D) = \frac{1}{\pi} \int_0^{\pi/2} \prod_{k=1}^{\delta_H} \left( 1 + \frac{D_k \gamma_s}{4\sin^2 \theta} \right)^{-n_R} d\theta,$$

(15)

where $D \equiv \{D_1, \ldots, D_{\delta_H}\}$ denotes the set of the nonzero squared Euclidean distances corresponding to a pairwise error event. If $D$ is given, we can obtain the exact value of the PEP through numerical integration of (15). However, to evaluate the FER, we should enumerate a number of sets of all distances of error events, which requires a large amount of memory and computation. Hence it is useful to determine an upper bound of the PEP that depends only on the product distance $\delta_p \equiv \prod_{k=1}^{\delta_H} D_k$, i.e., the product of all squared Euclidean distances.\footnote{As can be seen in (15), the PEP depends only on $\delta_p$, not on individual $D_k$’s, at high SNR.}

Recently, Byun et al. [3] presented a new, tight PEP upper bound of the form

$$P_B(\delta_H, \delta_p) = J(\delta_Hn_R) \left( \frac{\delta_p^{1/\delta_H} \gamma_s}{4} \right),$$

(16)

where

$$J_m(x) \equiv \left[ \psi(x) \right]^m \sum_{k=0}^{m-1} \binom{m-1+k}{k} [1 - \psi(x)]^k$$

(17)

for the positive integer $m$ and $\psi(x) \equiv \frac{1}{2} \left( 1 - \sqrt{1+x} \right), \ x \geq 0$\footnote{This new bound depends only on $\delta_p$ and turned out uniformly tighter than the Fitz bound in [2] and the tightest for a given $\delta_p$ [3].}

B. FER Upper Bound

The frame error probability of a coded system can be expressed by

$$P(e) = \sum_c P(c) P(e|c),$$

(18)

where $P(c)$ denotes the probability of transmitting codeword $c$, and $P(e|c)$ the frame error probability for a given transmitted codeword $c$, respectively. We can get an upper bound of $P(e|c)$ using the simple union bound of the form

$$P(e|c) \leq L \cdot \sum_{D} a(D|c) P_p(D),$$

(19)

where $a(D|c)$ denotes the number of first error events whose set of distances is equal to $D$, and $P_p(D)$ the pairwise error probability of such error event of the form in (15). A first error event is defined as an error event leaving the path $c$ at time 0 and remerging to $c$ after some number of trellis steps. In (19), the factor $L$ accounts for the number of time-shifted versions of each pairwise error event.\footnote{As seen in (18), $\delta_Hn_R$ and $\delta_p^{1/\delta_H}$ correspond to the diversity advantage and the coding advantage, respectively [1].}

Combining (18) and (19) we get the inequality

$$P(e) \leq L \cdot \sum_{D} a(D) P_p(D),$$

(20)

where $a(D) \equiv \sum_c P(c) a(D|c)$ denotes the average number of error events whose set of distances is equal to $D$. This inequality enables us to compute an FER upper bound through single numerical integration as long as the sets $D$’s and the corresponding weights $a(D)$’s are available.

Unfortunately, the evaluation of (20) requires the enumeration of a large number of sets $D$’s corresponding to first error events, which necessitates a large amount of memory and computation. In addition, it requires a numerical integration, since (20) is not a closed-form equation. However, if we take advantage of the FEP upper bound in (16), we can get an FER upper bound that depends only on the product distance $\delta_p$, not on the individual distances corresponding to the error events. The resulting expression is

$$P(e) \equiv L \cdot \sum_{\delta_H} \sum_{\delta_p} a(\delta_H, \delta_p) P_B(\delta_H, \delta_p),$$

(21)

where $a(\delta_H, \delta_p)$ denotes the average number of first error events whose symbol-wise Hamming distance and product distance are $\delta_H$ and $\delta_p$, respectively.

The FER upper bound in (21), in contrast to that in (20), takes a closed form. The error events can be sorted according to the symbol-wise Hamming distance $\delta_H$ and the product distance $\delta_p$, which reduces the required memory size. As to the enumeration of possible distance pairs $\langle \delta_H, \delta_p \rangle$’s and their
weights $a(\delta_H, \delta_p)$ we separate the analysis into two sections so that we can make detailed discussions on the distance spectrum issues and the computation reduction mechanisms.

IV. DISTANCE SPECTRUM OF SPACE-TIME CODES

Before discussing the enumeration issues of the distance pairs and their weights, we discuss the distance spectrum issues of space-time codes in fast fading channels. We first consider the distance spectrum of space-time codes in fast fading channels and examine the error state diagram to compute the distance spectrum. Then we discuss how to reduce the computation complexity of distance spectrum by exploiting the symmetric properties of the error state diagram.

A. Distance Spectrum in Fast Fading Channels

The evaluation of the FER union bound, in general, involves the computation of the distance spectrum of the code [13]. For space-time codes in fast fading channels, this “distance” may be interpreted to be the symbol-wise Hamming distance and the product distance, i.e., $(\delta_H, \delta_p)$. In this case the distance spectrum corresponds to the enumeration of the distance pair $(\delta_H, \delta_p)$ of every first error event and the associated weight $a(\delta_H, \delta_p)$. Here, the weight $a(\delta_H, \delta_p)$ is a measure of the relative frequency of occurrence of the pairwise error event with the distance pair $(\delta_H, \delta_p)$ in the corresponding union bound. The weight takes the expression

$$
a(\delta_H, \delta_p) = \frac{1}{2^R} \sum_{L_e \in L(\delta_H, \delta_p)} n(\delta_H, \delta_p; L_e) (2^R)^{-L_e}, \tag{22}
$$

where $2^R$ and $R$ respectively denote the number of states and the number of information bits corresponding to an edge in the trellis of the space-time code, $L(\delta_H, \delta_p)$ the set of lengths of the first error events with the distance pair $(\delta_H, \delta_p)$, and $n(\delta_H, \delta_p; L_e)$ the number of the first error events with distance pair $(\delta_H, \delta_p)$ and length $L_e$. The enumeration of the distances and the associated weights can be done by adapting the error state diagram to the fast fading channel case, which we will describe in the following.

B. Error State Diagram

The enumeration of the possible error events, in general, can be done by using an error state diagram [14], [5]. The error state diagram is formed by jointly attacking the transmitter state, $\sigma$, and the decoder state, $\hat{\sigma}$, at the same trellis stage into a product state ($\sigma = p, \hat{\sigma} = q$), $p, q = 0, \cdots, 2^R - 1$. A product state in an error state diagram is labeled as good if $\sigma = \hat{\sigma}$ and bad otherwise. A simple error event of length $L_e$ is defined as a path through the error state diagram in which a transition occurs from a good state to a bad state at some trellis stage. The error event consists of transitions among bad states for $L_e = 1$ trellis stages, and then a transition to a good state. For a $2^R$-state code with $R$ information bits per trellis stage, the total number of product states is $2^{2R}$ and from each product state there will be $2^R$ transitions. Among the $2^{2R}$ product states, $2^R$ states are good.

For space-time codes in fast fading channels, the distance metric that needs to be enumerated using an error state diagram is the distance pair $(\delta_H, \delta_p)$. We can compute this distance metric iteratively along the error state diagram in the following manner: Let $c_t$ and $\hat{c}_t$ be the transmitted and the decoded space-time symbols at trellis stage $t$, respectively. Then, we can update the distance metric at trellis stage $t$, $(\delta_H(t), \delta_p(t), t = 1, 2, \cdots, L_e)$, iteratively by considering the following two cases.

First, if $c_t = \hat{c}_t$, then

$$
\delta_H(t) = \delta_H(t-1),
\delta_p(t) = \delta_p(t-1). \tag{23}
$$

Secondly, if $c_t \neq \hat{c}_t$, then

$$
\delta_H(t) = \delta_H(t-1) + 1,
\delta_p(t) = \delta_p(t-1) \times \|c_t - \hat{c}_t\|^2. \tag{24}
$$

For this iterative computation, the distances are initialized with $\delta_H(0) = 0$ and $\delta_p(0) = 0$.

For example, we consider the case of four state, $R = 1$, $n_T = 2$ BPSK code of Hammons [15], having the generator matrix

$$
G_1 = \begin{bmatrix} 1 & 0 & 1 \\
1 & 1 & 1 \end{bmatrix}.
$$

Fig. 1 depicts the encoder structure, the trellis diagram, and the constellation mapping of this space-time code. In the trellis diagram in Fig. 1 (b), the solid line represents the transition by the input ‘0’ and the dashed line the transition by the input ‘1’. The branch label represents the encoder outputs, i.e., $f^1_t$ and $f^2_t$ in (7).

The error state transition table of the space-time code is given in Table I. Each row of the table is determined by the corresponding product state $(\sigma, \hat{\sigma})$ and each column is determined by the corresponding transmitted and decoded input bit pair $(a^1_t, a^2_t)$. Among the 16 product states, $\{\sigma_1, \sigma_6, \sigma_{11}, \sigma_{16}\}$ is the set of good states. Table entry $[(\sigma, \hat{\sigma}), (b^1_H, b^2_H)]$ denotes the next product state and the elements needed for the update of the distance metric, which is done by the relation

$$
\delta_H(t) = \delta_H(t-1) + b^1_H,
\delta_p(t) = \delta_p(t-1) \times b^2_p. \tag{25}
$$

For example, if the current product state is $\sigma_1 \equiv (0, 0)$ and the input is $(0, 0)$, then the next product state is $\sigma_1$ and the transmitted and the decoded space-time symbols are $c_t = \hat{c}_t = [1 1]^T$. So the distance metric is updated according to (23), or, equivalently, $(b^1_H, b^2_H) = (0, 1)$. For another example, if the current product state is $\sigma_2 \equiv (1, 0)$ and the input is $(0, 1)$, then the next product state is $\sigma_7 \equiv (2, 1)$ and $c_t = [1 - 1]^T$, $\hat{c}_t = [-1 1]^T$. So $c_t \neq \hat{c}_t$ and $\|c_t - \hat{c}_t\|^2 = 4$ and hence the distance metric is updated according to (24), or, equivalently, $(b^1_H, b^2_H) = (1, 4)$.

C. Complexity Reduction

For geometrically uniform codes [16], the distance spectrum can be computed using an error state diagram constructed
with the assumption that the all-zero path is transmitted. In this case, the number of states in the error state diagram is reduced to $2^{n_t}$. However, space-time codes are not geometrically uniform in general [1], so the entire error state diagram needs to be considered. Therefore, to compute the distance spectrum, we should consider all the paths from each good product state to each good product state, which would be a considerable burden of memory space and computation. We exploit the symmetry of the error state transition table to reduce the complexity.

From the rows corresponding to the good product states in Table I, we can observe that the element $(b_{ij}, b_p)$ which is related to the update of the distance metric depends only on the input, not on the current product state. Generalizing this idea, we can establish the following theorem.

**Theorem 1:** In space-time trellis coded MPSK modulation, the distance metric $(\delta_H, \delta_p)$ of an error event depends only on the input $(a^k, \hat{a}^k)$, $t = 1, \ldots, L_e$, $k = 1, \ldots, R$, but not on the starting good product state, for the length of the error event, $L_e$, and the number of input bits per state transition, $R$.

**Proof:** Let $f_t^i$ and $\hat{f}_t^i$ be the transmitted and the decoded encoder outputs for $t = 1, \ldots, L_e$ and $i = 1, \ldots, n_T$. Then, from (7), we get

$$f_t^i = \sum_{k=1}^{R} \sum_{l=0}^{\nu_k} g_{ij}^k a_{t-l}^k \mod M,$$

$$\hat{f}_t^i = \sum_{k=1}^{R} \nu_k g_{ij}^k \hat{a}_{t-l}^k \mod M.$$

We can assume $a_t^k = \hat{a}_t^k$ for $t \leq 0$ as the product state at stage $t = 0$ is good. So if we separate out the contribution of the starting product state from that of the input, we get

$$f_t^i = C + \sum_{k=1}^{R} \sum_{l=1}^{t} g_{ij}^k a_{t-l}^k \mod M,$$

$$\hat{f}_t^i = C + \sum_{k=1}^{R} \nu_k g_{ij}^k \hat{a}_{t-l}^k \mod M,$$

where $C \equiv \sum_{k=1}^{R} \sum_{l=1}^{t} g_{ij}^k a_{t-l}^k$ is a constant that depends on the starting product state. Based on this, we can evaluate the Euclidean distance between the transmitted and the decoded space-time symbols as follows:

$$|c_t^i - \hat{c}_t^i|^2 = \exp\left(\frac{2\pi f_t^i}{M}\right) - \exp\left(\frac{2\pi \hat{f}_t^i}{M}\right)^2 \exp\left(\frac{2\pi}{M} \sum_{k=1}^{R} \nu_k g_{ij}^k \hat{a}_{t-l}^k \right)^2.$$

In the equation $|c_t^i - \hat{c}_t^i|^2$ depends only on $(a_t^k, \hat{a}_t^k)$, not on $C$ which is related to the starting product state. But the distance metric $(\delta_H, \delta_p)$ of an error event is a function of $|c_t^i - \hat{c}_t^i|^2$ for $t = 1, \ldots, L_e$. Therefore $(\delta_H, \delta_p)$ depends only on the input, not on the starting product state. This proves the theorem.

By Theorem 1, we can compute the distance spectrum of space-time codes in fast fading channels considering only the paths that have $(0, 0)$ as the starting good product state. This

![Fig. 1. Hammons 4-state BPSK code: (a) Encoder, (b) trellis diagram, (c) constellation mapping.](image-url)
helps to reduce the number of paths to consider by the factor of $2^\nu$ (i.e., the factor of the number of good product states), which reduces greatly the amount of memory and computation required.

In the case of BPSK space-time codes, a property of stronger symmetry holds. The distance metric of an error event depends only on the difference of the transmitted and the decoded inputs, $\hat{a}_t^k \oplus \hat{a}_t^{k'}$, $t = 1, \cdots, L_e$, $k = 1$, for the length of the error event, $L_e$. \footnote{\(\oplus\) denotes the bit-wise exclusive OR operation.}

**Theorem 2:** In space-time trellis coded BPSK modulation, the distance metric \((\delta_H, \delta_p)\) of an error event depends only on the difference of the transmitted and the decoded inputs, $\hat{a}_t^k \oplus \hat{a}_t^{k'}$, $t = 1, \cdots, L_e$, $k = 1$, for the length of the error event, $L_e$. \footnote{In \cite{5}, Aktas and Fitz proposed a distance spectrum computation algorithm for space-time codes in slow fading channels. We have adapted it to the fast fading channels to obtain the algorithm introduced in this section.}

**Proof:** We can evaluate the Euclidean distance between the transmitted and the decoded space-time symbols based on (26) as follows:

$$
|c_t^i - \hat{c}_t^i|^2 = \exp \left( j \frac{2\pi}{M} \sum_{l < t} g_{l,i}^1 a_{l-t}^i \right) - \exp \left( j \frac{2\pi}{M} \sum_{l < t} g_{l,i}^1 \hat{a}_{l-t}^i \right)^2
$$

$$
= 1 - \exp \left\{ j \frac{2\pi}{M} \sum_{l < t} \left[ g_{l,i}^1 \left( \hat{a}_{l-t}^i - a_{l-t}^i \right) \right]\right\}^2.
$$

(27)

In the equation $|c_t^i - \hat{c}_t^i|^2$ depends on $\hat{a}_t^i - a_t^i$ for $t = 1, \cdots, L_e$. In the case of BPSK ($M = 2$), $\hat{a}_t^i - a_t^i$ may be replaced with $\hat{a}_t^i \oplus a_t^i$ without changing $|c_t^i - \hat{c}_t^i|^2$, since $\exp \left( j \frac{2\pi}{M} \right) = -1 = \exp \left( -j \frac{2\pi}{M} \right)$. Hence, in the BPSK space-time codes, we get

$$
|c_t^i - \hat{c}_t^i|^2 = 1 - (-1) \sum_{l < t} g_{l,i}^1 \left( \hat{a}_{l-t}^i \oplus a_{l-t}^i \right)^2,
$$

(28)

which proves that the distance metric $(\delta_H, \delta_p)$ depends only on $a_t^i \oplus \hat{a}_t^i$, $t = 1, \cdots, L_e$.

By Theorem 2, BPSK space-time codes become geometrically uniform with respect to the distance metric $(\delta_H, \delta_p)$. So, it is possible to compute the distance spectrum of BPSK space-time codes in fast fading channels assuming that the all-zero path is transmitted. Therefore the number of product states reduces to $2^\nu$. This enables an additional saving in computation.

**V. COMPUTATION OF DISTANCE SPECTRUM**

Now, we introduce a new algorithm to compute the distance spectrum for space-time codes in fast fading channels.\footnote{\cite{5} to \cite{8} proposed a distance spectrum computation algorithm for space-time codes in slow fading channels. We have adapted it to the fast fading channels to obtain the algorithm introduced in this section.}

The distance spectrum search is based on the fact that the symbol-wise Hamming distance $\delta_H$ is a monotonically nondecreasing function of time, i.e., $\delta_H(t) \leq \delta_H(t+1)$. We compute the truncated distance spectrum for $\delta_H \leq H$. $H$ is an arbitrarily chosen threshold value for the number of terms to include in the distance spectrum. Fig. 2 shows the flow chart of the distance spectrum computation algorithm that we describe below.

To begin with, we initialize the unterminated path list at time 0. By Theorem 1, we may consider only one good product state, without loss of generality, so we take the good product state $(0,0)$ as the only element in the unterminated path list at time 0. We also initialize the time index $t$ to 0. Then, we select a path from the unterminated path list at time $t$, and select a one-stage extended path from the path. As the number of possible inputs $(a_{t+1}^k, \hat{a}_{t+1}^k)$ is $2^{2R}$, the number of all possible transitions from a given state is $2^{2R}$, which implies that the number of extended paths from a path is $2^{2R}$. In the case of BPSK space-time codes, the transmitted codeword may be assumed to be the all-zero codeword by Theorem 2, so the number of extended paths from a path reduces to $2^R = 2$. Next, we check if the extended path reaches a good state. If it does, we add this path to the terminated path list in case $\delta_H(t) \leq H$ and eliminate this path in case $\delta_H(t) > H$. Otherwise, we add this path to the unterminated path list at time $t+1$ in case $\delta_H(t) \leq H$ and eliminate this path in case $\delta_H(t) > H$. We repeat this procedure for all extended paths from all unterminated paths at time $t$. Then, we check if the...

![Fig. 2. Flow chart of the distance spectrum computation algorithm.](image-url)
unterminated path list at time $t+1$ is empty. If so, we end the algorithm, as it means that we have enumerated an exhaustive set of the terminated paths satisfying $\delta_H \leq H$. Otherwise, we increase the time index $t$ by 1 and repeat the above operation of sequential selection and extension at the next stage.

When adding a path to the un terminated path list or to the terminated path list, we record the length of the path, $L_c$, as well as the distance metric $(\delta_H, \delta_p)$, since the weight in the distance spectrum depends also on $L_c$ as can be seen in (22). After completing the above operations, we sort the terminated paths with respect to the distance metric $(\delta_H, \delta_p)$ and compute the associated weight $\hat{a}(\delta_H, \delta_p)$ as in (22).

We can conceive that if it is guaranteed that the symbol-wise Hamming distance $\delta_H$ increases at the last remerging stage of each error event, then we may exclude many un terminated paths whose symbol-wise Hamming distances have already reached $H$ in the distance spectrum algorithm. In Table I, we can observe that $b_H$ always takes on 1 for the elements corresponding to all transitions from bad states to good states. Generalizing this idea, we can establish the following theorem.

Theorem 3: Let $g^k_i = [g^k_{v_1}, g^k_{v_2}, \ldots, g^k_{v_n}]^T$ be an $n_T \times 1$ vector containing the rightmost column of the generator matrix $G_k$, and $e^k \in \{-1, 0, 1\}$ for $k = 1, \ldots, R$. If

$$\sum_{k=1}^{R} e^k \cdot g^k_{v_k} = 0 \mod M$$

implies \(^7\)

$$e^1 = \cdots = e^R = 0,$$

then the symbol-wise Hamming distance $\delta_H$ always increases at the last remerging stage of each error event.

Proof: Let $L_c$ be the time index corresponding to the last stage of an arbitrary error event. It suffices to show that $c_{L_c} \neq \hat{c}_{L_c}$ in case (29) implies (30). Since the product state at time $L_c$ is good, we get

$$a^k_{L_c-1} = \hat{a}^k_{L_c-1}, \quad 0 \leq l \leq \nu_k - 1, \quad 1 \leq k \leq R. \quad (31)$$

The Euclidean distance between the transmitted and the decoded space-time symbols at time $L_c$ becomes

$$||c_{L_c} - \hat{c}_{L_c}||^2 = \sum_{i=1}^{n_T} |c^i_{L_c} - \hat{c}^i_{L_c}|^2$$

$$= \sum_{i=1}^{n_T} \left| 1 - \exp \left\{ j \frac{2\pi}{M} \sum_{k=1}^{R} \sum_{l=0}^{\nu_k} g^k_{v_i} \left( \hat{a}^k_{L_c-l} - a^k_{L_c-l} \right) \right\} \right|^2, \quad (32)$$

by (27), turning into

$$||c_{L_c} - \hat{c}_{L_c}||^2 = \sum_{i=1}^{n_T} \left| 1 - \exp \left\{ j \frac{2\pi}{M} \sum_{k=1}^{R} g^k_{v_i} \left( \hat{a}^k_{L_c-v_k} - a^k_{L_c-v_k} \right) \right\} \right|^2, \quad (33)$$

by (31).

\(^7\)0 denotes an all-zero vector.

We prove that $c_{L_c} \neq \hat{c}_{L_c}$ by contradiction. If $c_{L_c} = \hat{c}_{L_c}$, we get

$$\sum_{k=1}^{R} g^k_{v_i} \left( \hat{a}^k_{L_c-v_k} - a^k_{L_c-v_k} \right) = 0 \mod M, \quad 1 \leq i \leq n_T,$$

which is equivalent to (29) for $e^k = \hat{a}^k_{L_c-v_k} - a^k_{L_c-v_k}$. By assumption, (29) implies (30), that is,

$$a^k_{L_c-v_k} = \hat{a}^k_{L_c-v_k}, \quad 1 \leq k \leq R. \quad (34)$$

By (31) and (34), we find that the product state at time $L_c - 1$ is also good. However this contradicts to the fact that the transition from a bad state to a good state occurs at time $L_c$, the last stage of the error event. Therefore, if (29) implies (30), then $c_{L_c} \neq \hat{c}_{L_c}$ and hence the symbol-wise Hamming distance increases at time $L_c$. This completes the proof.

By Theorem 3, we can exclude all un terminated paths that have the symbol-wise Hamming distance of $H$ in the distance spectrum computation for the space-time code satisfying the property that (29) implies (30). This helps to reduce a large amount of memory space and computation time again. In this case, the upper limit of $\delta_H$ for the record of the un terminated paths decreases from $H$ to $H - 1$ (see Fig. 2). Also the block for checking if $\delta_H \leq H$ for the record of the terminated paths (i.e., the block enclosed by dashed rectangle in Fig. 2) is not necessary and all the terminated paths should be recorded.

In general, for most well-designed space-time codes, (29) implies (30). Hammongs’s BPSK space-time code considered in the previous section also retains this property. It’s straightforward to show that in Theorem 3 if $g^k_{v_1}$ is substituted by $g^k_{v_1}$, which contains the leftmost column of $G_k$, then $\delta_H$ always increases at the first diverging stage of each error event. Based on these facts, we can establish a simple criterion in designing good space-time codes in fast fading channels: The leftmost columns of the generator matrices, as well as the rightmost columns, should be linearly independent modulo $M$, with respect to the coefficients in $\{-1, 0, 1\}$, for M-PSK space-time codes.

VI. NUMERICAL RESULTS

We apply the above results to space-time trellis-coded modulation (TCM) schemes. We use the newly derived FER upper bound to evaluate the performances using distance spectrum, comparing them with the simulation results. In evaluating analytical bounds, we use the truncated distance spectrum of threshold $H$, which implies that the FER upper bound may be approximated satisfactorily by considering only the distance metrics for $\delta_H \leq H$ in the distance spectrum. We also evaluate the code FER performances through simulations. For the simulations, we take 130 symbols per frame (i.e., $L = 130$) for $10^{12}$ frames and employ a maximum likelihood Viterbi decoder with perfect CSI at the receiver. We plot the resulting FER performance curves with respect to the symbol SNR per receive antenna, $n_T E_s/N_0$.\(^8\)

\(^8\)Linear independence of $g^k_{v_1}$’s means that (29) implies (30) for $e^k \in \{-1, 0, 1\}$.\(^2\)
**TABLE II**

**DISTANCE SPECTRUM OF TAROKH 4-STATE QPSK SPACE-TIME CODE.**

<table>
<thead>
<tr>
<th>$D$</th>
<th>${2,2}$</th>
<th>${4,4}$</th>
<th>${2,2,4}$</th>
<th>${2,4,6}$</th>
<th>${4,4,8}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a(D)$</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$\delta_H$</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$\delta_P$</td>
<td>4</td>
<td>16</td>
<td>16</td>
<td>48</td>
<td>128</td>
</tr>
</tbody>
</table>

**TABLE III**

**DISTANCE SPECTRUM OF VUCETIC 16-STATE QPSK SPACE-TIME CODE.**

<table>
<thead>
<tr>
<th>$\delta_H$</th>
<th>3</th>
<th>3</th>
<th>3</th>
<th>4</th>
<th>4</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_P$</td>
<td>64</td>
<td>96</td>
<td>128</td>
<td>128</td>
<td>192</td>
<td>384</td>
</tr>
<tr>
<td>$a(\delta_H, \delta_P)$</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

(Example 1) We compare the FER upper bound in (20) that employs the exact PEP $P_P(D)$ in (15) and considers the sets $D$’s corresponding to the error events, with the FER upper bound in (21) that employs the PEP upper bound $P_B(\delta_H, \delta_P)$ in (16) and considers only $\delta_H$ and $\delta_P$. We consider the case of four state, $R = 2$, $n_T = 2$ QPSK code of Tarokh [1], having the generator matrices

$$G_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}. \quad \quad \quad \quad \quad \quad \quad (3)$$

This code provides a minimum symbol-wise Hamming distance of 2. The bounds are approximated by considering all error events for $\delta_H \leq 3$ in the distance spectrum (i.e., $H = 3$). Then the sets $D$’s and the associated distance metrics $(\delta_H, \delta_P)$’s are as given in Table II. These values can be obtained through the distance spectrum calculation algorithm given in Section V. There are two types of error events of the symbol-wise Hamming distance 2 and three types of error events of the symbol-wise Hamming distance 3. Fig. 3 (a)–(c) show the FER for this space-time code with one, two, and four receive antennas, respectively. From the figures, we observe that the FER bound employing the exact PEP and that employing the PEP upper bound nearly coincide, which confirms that the PEP upper bound is sufficiently tight.

(Example 2) We compare the derived analytical FER bounds and the simulation results by considering the case of sixteen state, $R = 2$, $n_T = 2$ QPSK code of Vucetic [12], [17], having the generator matrices

$$G_1 = \begin{bmatrix} 0 & 0 & 2 \\ 2 & 1 & 2 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 2 \end{bmatrix}. \quad \quad \quad \quad \quad \quad \quad (4)$$

This code provides a minimum symbol-wise Hamming distance of 3. The bounds are approximated by considering all error events for $\delta_H \leq 4$ in the distance spectrum. The distance spectrum of this space-time code is given in Table III. Only the distance metrics $(\delta_H, \delta_P)$’s, rather than the sets $D$’s, are considered in this case. There are three types of error events of the symbol-wise Hamming distance 3 and six types of error events of the symbol-wise Hamming distance 4. Fig. 4 (a)–(c) show the FER for this space-time code with one, two, and four receive antennas.

The distance spectrum in Table II is identical to that in [9] which is computed assuming that the all-zero codeword is transmitted. This happens because of the geometrical uniformity of the Tarokh 4-state QPSK space-time code. The results differ in general.
four receive antennas, respectively. The analytical bounds for \( H = 4 \) are close to the simulation results. The bound curve for \( H = 3 \) lies beneath the simulation curve, which implies that the effect of the error events of minimum symbol-wise Hamming distance on the FER performance is not dominant and the error events of non-minimum symbol-wise Hamming distance should also be considered.

(Example 3) We consider the case of eight state, \( R = 2 \), \( n_T = 2 \) QPSK code of Bäro [18], having the generator matrices

\[
G_1 = \begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix}.
\]

This code provides a minimum symbol-wise Hamming distance of 2. The bounds are approximated by considering all error events for \( \delta_H \leq 4 \). The distance spectrum of this space-time code is given in Table IV. There are one type of error event of the symbol-wise Hamming distance 2, six types of error events of the symbol-wise Hamming distance 3, and nine types of error events of the symbol-wise Hamming distance 4. Fig. 5 (a)–(c) show the FER for this space-time code with one, two, and four receive antennas, respectively. The bound curve for \( H = 2 \) lies beneath the simulation curve. Similarly to the case of the previous example, we can see that the performance is not dominated by the error events of minimum symbol-wise Hamming distance. The analytical bounds for \( H = 4 \) and \( H = 3 \) are close to the simulation results. The effect of error events for \( \delta_H = 4 \) on the performance is less significant than that of error events for \( \delta_H = 3 \).

VII. CONCLUDING REMARKS

We have analyzed the performance and introduced the distance spectrum computation method of the space-time codes in fast Rayleigh fading channels. In support of this, we have derived a new FER upper bound that is very accurate and simple to compute, and also have devised a simplified distance spectrum computation method with reduced complexity.

The PEP is a function of all Euclidean distances of the pairwise error event. So, in order to exactly evaluate the FER union bound, a number of sets containing all Euclidean distances of error events should be enumerated, which requires a large amount of memory space and computation. We have alleviated this problem by taking advantage of the PEP upper bound recently proposed by the authors themselves which depends only on the product distances of error events. This PEP upper bound helps to reduce the computation burden significantly as it takes a closed-form expression, in contrast to
the exact PEP which requires numerical or residue integration to compute.

In the past, the performance analysis of space-time codes in fast fading channels used to be done under the assumption that the all-zero codeword is transmitted. However, this substantially degrades the accuracy as space-time codes are not geometrically uniform in general. We have chosen to consider all the cases of possible transmitted codewords to improve the accuracy. In order to get around the problem of computational complexity increase, however, we have exploited the symmetry of the error state diagram. We have established that it is sufficient to consider only one good product state as the starting state. In the case of BPSK space-time codes, we have shown that the all-zero codeword may be assumed to be transmitted without loss of generality.

Throughout the numerical examples considered, we have observed that the new FER upper bound is sufficiently tight in most practical cases. We can estimate the FER performances of space-time codes in fast fading channels very accurately by computing the distance spectrum and using the derived bound, without conducting time-consuming Monte-Carlo simulations. Generalizing the analysis technique introduced in this paper to incorporate antenna spatial and temporal correlations and other channel fading characteristics, and designing good space-time codes based on the derived FER bound or the distance spectrum, are avenues for further study.

REFERENCES


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