Analytic expression for the temperature of the current-heated nanowire for the current-induced domain wall motion

Chun-Yeol You,a In Mo Sung, and Byung-Kyu Joe
Department of Physics, Inha University, Incheon 402-751, Korea

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The authors find a simple analytic expression for the temperature of Joule heated nanowire by current pulse, which is important in the study of the current induced domain wall motion. Since the effect of spin transfer torque depends on the thermal energy of the system, the temperature of the nanowire is a vital information. Even though the numerical solution of the heat conduction equation is well established, not only does it require a lot of numerical effort, but neither does it give any physical insight. With appropriate assumptions and Green's function method, the author derive a simple expression for the temperature of the nanowire as a function of the current density, sample geometry, and thermal properties of the substrate. The authors confirm the validity of their analytic expression by the comparison between the results of a simple expression and a commercial finite element method. © 2006 American Institute of Physics. [DOI: 10.1063/1.2399441]

Since the theoretical predictions1 and experimental confirmations2 of the spin transfer torque, a lot of studies have been reported.3 5 The spin transfer torque related phenomena such as current induced magnetization switching6 and current induced domain wall movement7 10 (CIDWM) are promising technologies for the spintronics devices, but the experimental observation of the spin transfer torque effect requires a huge current density of 1010 1011 A/m2. Such a huge current density generates Joule heating of the system, and raising the temperature causes not only changes of spin dynamics due to the thermal energy,11 12 but also a breakdown of the system. According to a recent experiment,13 the effect of Joule heating is serious with the current density of >5.0 1011 A/m2, and even the wire was heated near the Curie temperature with current density of 7.5 1013 A/m2. Therefore, the temperature of the nanowire with Joule heating is a very important information for the analysis of the experimental data and design of the nanowire system.

The temperature profile can be obtained by solving the heat conduction equation. Solving heat conduction equation entails a lot of numerical work with finite element method (FEM).14 Nowadays, commercial software15 is available to solve the heat conduction equation with minimal effort. However, it is not physically transparent and any physical insight can be extracted from the numerical solutions. Green's function method16 is widely used in order to obtain the analytic solution of the heat conduction equation, and many remarkable successes have been reported in the optical disk community.17 18

In this letter, we solve the heat conduction equations by employing Green’s function method. With proper approximations, we obtain a simple expression for the temperature of the nanowire heated by current pulse. The validity of our analytic expression is confirmed by the comparison with the results of a commercial FEM software.15

The heat conduction equation is given by

$$\rho C \frac{\partial T(r,t)}{\partial t} = K \nabla^2 T(r,t) + S(r,t),$$

where \( \rho \), \( C \), \( K \), \( T(r,t) \), and \( S(r,t) \) are the density, specific heat, thermal conductivity of the medium, and temperature and heat source at the position of \( r \) and time \( t \), respectively. According to the procedures of Ref. 18, the temperature of the layered structure is given by

$$T(r,t) = \frac{1}{\rho C} \int_0^t \int_{\mathcal{V}} G(r,t; \mathcal{r}',t') S(\mathcal{r}',t') dV' dt'.$$

The appropriate Green’s functions that satisfies boundary conditions (\( \partial G(z=0)/\partial z = 0 \)) are as follows:

$$G(r,t; \mathcal{r}',t') = G_{xy}(s,t; s',t') G_{z}(z,t; z',t'),$$

$$G_{xy}(s,t; s',t') = \frac{1}{4\pi \mu(t-t')} \exp \left( -\frac{(x-x')^2 + (y-y')^2}{4\mu(t-t')} \right),$$

$$G_{z}(z,t; z',t') = \frac{1}{\sqrt{4\pi \mu(t-t')}} \left( \exp \left( -\frac{(z-z')^2}{4\mu(t-t')} \right) + \exp \left( -\frac{(z+z')^2}{4\mu(t-t')} \right) \right),$$

for the semi-infinite medium.19 Here, \( \mu_5 = K_3/\rho_5 C_5 \) is the diffusivity of substrate. In our case, the heat source is given by Joule heating due to the current flow in the wire, and the heat can flow only through the substrate as shown in Fig. 1(a). Here, we ignored the effect of convection at the surface of the nanowire due to its small surface area. The heat source term is written as

$$S(r,t) = S_y(s,t) S_A(z)p(t),$$

where \( S_y \) is a power density per unit area in the \( x-y \) plane by Joule heat, \( S_A \) is power absorption rate to the \( z \) axis per unit power, and \( p(t) \) is pulse function. Now, let us make the main assumptions. (i) The length of wire \( L \) is infinitely long, so we can reduce the problem to two-dimensions. (ii) The temperature of the nanowire is the same throughout the wire. Since typical diffusivity \( \mu \) of the ferromagnetic metal is of the...
order of $10^{-5}$ m$^2$/s, it means the thermal equilibrium is achieved within 1 $\mu$s for $\sim 1$ $\mu$m length scale. (iii) In the viewpoint of the substrate, the heat source exists only at the interface ($z=0$), as illustrated in Fig. 1(b). (iv) The temperature of the wire is the same as the temperature of the interface of the substrate. With above assumptions, $S_J, S_A$ and $\rho(t)$ are defined as follows:

$$S_J(t) = \delta(t), \quad p(t) = \theta(t) - \theta(t-t_p).$$  

(7)

$$S_{JG}(x,t) = \frac{P_0}{\sqrt{\pi}w_G L} \exp\left(-\frac{x^2}{w_G^2}\right),$$

(8)

$$S_{JH}(x,t) = \frac{P_0}{w_G L} \left(\theta\left(x+\frac{w}{2}\right) - \theta\left(x-\frac{w}{2}\right)\right).$$

(9)

Here, $\delta(t)$ is the Dirac-delta function, $\theta(x)$ is a step function, $t_p$ is the current pulse duration time, $w$ is the width of the wire, $w_G=\alpha w$ ($\alpha \sim 0.5$) is the width of the Gaussian profile, and $\alpha$ is an adjustable parameter, which will be determined later. The Joule heat due to current density $J$ is $P_0 = RF^2 = L(whJ)^2/\sigma_w w h = L(whJ)^2/\sigma_w$. Here, $\sigma_w, h, w$ are the electric conductivity, thickness, and width of the wire, respectively. $S_{JG}$ and $S_{JH}$ are power density profiles of Gaussian and rectangle shapes of width $w$, as illustrated in Figs. 1(c) and 1(d). $S_{JG}$ is less realistic, but mathematical handling is easy and the final result is simpler, while $S_{JH}$ is more realistic power distribution, but the final result is more complicated.

The temperature profile for a given position $r$ and time $t$ is determined by using Eq. (2):

$$T(r,t) = \int_0^t U(x,\tau)V(z,\tau)d\tau - \theta(t-t_p) \int_{-t_p}^0 U(x,\tau)V(z,\tau)d\tau,$$

(10)

$$U(x,\tau) = \frac{1}{\rho_s C_s} \int_{-\infty}^\infty \int_{-\infty}^\infty G_{s}(s,t|s',t') S_J(x,t') dx' dy' \times S_J(x,t') dx' dy'$$

(11)

$$V(z,\tau) = \int_{-\infty}^\infty G_{s}(z,t|z',t') S_A(z,t') dz'$$

(12)

where $\tau = t-t'$, and $\rho_s$ and $C_s$ are the density and specific heat of the substrate. By substitution Eqs. (7)–(12):

$$U^G(x,\tau) = \frac{P_0}{L \sqrt{\pi} (w_G^2 + 4\mu_s \tau)} \exp\left(-\frac{x^2}{w_G^2 + 4\mu_s \tau}\right),$$

(13)

$$U^H(x,\tau) = \frac{P_0}{2wL} \left[\exp\left(\frac{w + 2x}{4\mu_s \tau}\right) + \exp\left(\frac{w - 2x}{4\mu_s \tau}\right)\right],$$

(14)

$$V(z,\tau) = \frac{1}{\sqrt{\pi} \mu_s} \exp\left(-\frac{z^2}{4\mu_s \tau}\right).$$

(15)

The erf($x$) is an error function. Now, the remaining process is the integration of Eq. (10). Unfortunately, there is no analytic solution of the integral in Eq. (10). If we need the temperature profiles of the substrate, we can numerically integrate Eq. (10). Here, what we really want to know is not the whole temperature profile, but the temperature of the wire itself, which is the same as the temperature of the substrate interface. Therefore what we need is to find the temperature at the origin, $T(x=0, z=0, t)$, and in that case, the integral of Eq. (10) is easily done:

$$T^G(t) = \frac{whJ^2}{\pi \mu_s \sigma_w \rho_s C_s} \left\{\text{arcsinh}\left(\frac{2\sqrt{\mu_s t}}{w_G}\right) - \theta(t-t_p) \text{arcsinh}\left(\frac{2\sqrt{\mu_s (t-t_p)}}{w_G}\right)\right\},$$

(16)

$$T^H(t) = TR(t) - \theta(t-t_p) TR(t-t_p),$$

where

$$TR(t) = \frac{2hJ^2}{\pi \sigma_w \rho_s C_s} \left[\frac{\pi}{\mu_s} \text{erf}\left(\frac{w}{4\mu_s t}\right) + \frac{2hJ^2}{2\pi \sigma_w \rho_s C_s} \Gamma\left(0, \left(\frac{w}{4\mu_s t}\right)^2\right)\right],$$

(17)

where $\Gamma$ is the incomplete Gamma function.

By the comparisons of the leading term of the Taylor series of $T^G(t)$ and $T^H(t)$ for $t \gg w/\mu_s$, we find $\alpha = 0.5$, it seems reasonable due to the geometrical meaning of $w_G$. Equation (16) is written in simpler form for $t \gg w/\mu_s$ as follows:

$$T^G(t) = \frac{whJ^2}{\pi \mu_s \sigma_w \rho_s C_s} \ln\left(\frac{4\mu_s t}{w_G}\right) - \theta(t-t_p) \ln\left(\frac{4\sqrt{\mu_s (t-t_p)}}{w_G}\right).$$

(18)

Equation (18) is the central result of this letter. Figure 2 shows comparisons between Eqs. (10) and (16)–(18) with FEM calculation result. Here, we considered the wire structure of Ref. 13, $w=240$ nm, $h=10$ nm, conductivity of FeNi.
is taken as that of Ni [$\sigma_w=1.38 \times 10^7$ (Ωm)$^{-1}$], and the thermal parameters of SiO$_2$ are used for the substrate ($\mu_S=8.27 \times 10^{-7}$ m²/s, $C_S=730$ J/kg/K, $K_S=1.4$ W/m/K, and $\rho_S=2200$ kg/m³), with $t_p=5 \mu$s and $J=10^{12}$ A/m². All Green’s function results are indistinguishable, and we confirmed that the FEM result shows that the temperatures of the whole wire area and the interface are the same as we assumed (not shown). The FEM data show qualitatively the same result, however, there is a small discrepancy between Green’s function method and FEM. We believe that the discrepancy came from our third assumption. We assumed the delta function like heat source, but it is not true in real system. Therefore the discrepancy is smaller for thinner case. In order to check the validity of our result, we compared peak temperatures at $t=t_p$ in Fig. 3. The FEM results also show linear dependence on $h$ as $T(t=t_p)$ with slightly different slope. Our analytic expression is quite good for a few tens of nanometer thick wire. Since the slope of the line depends on the adjustable parameter $\alpha$, we find the proper value of $\alpha=0.605$ to fit the FEM result.

It must be pointed out that the experiment data of Yamaguchi et al. showed more serious Joule heating in the same condition. Even though we ignored convection effect, which practically reduces the temperature, our result gives a much smaller temperature increment compared to the experimental data. The possible reasons for the inconsistency are as follows: The conductivity of the wire is decreased, causing the increase of the temperature, which is not considered in our study; and the material parameters used are taken from literature, probably different from the values in the experiment.

In this letter, we find a simple analytic expression for the temperature of the nanowire heated by the current. The dependences of the wire temperature on the physical parameters of the wire and substrate materials, and geometrical configurations such as wire width and thickness are explicitly shown. Our results give a physical insight on the design of nanowires for CIDWM experiments, and information on the temperature can be easily extracted from the given experimental condition, which is important for proper theoretical interpretation.

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15. For example, COMSOL MULTIPHYSICS, http://www.comsol.com
19. We need only a semi-infinite medium in this study. For finite thickness medium, Green’s function is given in Ref. 18.
20. For small $t\approx t_p$, the contribution of the second term is very small, so Eq. (19) is quite valid for large $t$. 

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**FIG. 2.** Temperature variations with $t_p=5 \mu$s pulse. Equations (10) and (16)–(18), are solid lines, and indistinguishable. FEM result is □.

**FIG. 3.** Peak temperature at $t=t_p$. FEM result is □. Eq. (18) with $\alpha=0.5$ is a solid line, and $\alpha=0.605$ is a red dashed line.
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