Performance Analysis of Multiuser Diversity under Transmit Antenna Correlation

Daeyoung Park, Member, IEEE, and Seung Young Park, Member, IEEE

Abstract—In this paper, we investigate the effect of spatial correlation on throughput performance of downlink multi-antenna transmission schemes exploiting multiuser diversity, in which partial channel information such as signal-to-interference plus noise power ratio (SINR) is available at the transmitter. The asymptotic analysis is performed based on the extreme value theory. From this analysis, we demonstrate that the throughput optimal transmission scheme depends on the degree of the antenna correlation and the operating SNR. Especially, the multiuser spatial multiplexing known as the asymptotically optimal transmission scheme is no longer optimal in highly correlated multiple antenna channels.

Index Terms—Multiuser diversity, MIMO, antenna correlation.

I. INTRODUCTION

WIRELESS packet scheduling has attracted much attention because the multiuser diversity gain yields high spectral efficiency in multiuser wireless networks. This gain stems from the fact that there is at least one user whose channel is near peak when the number of users is high and its channel varies independently [1]. In this context, randomness in point-to-multipoint communications is a factor to be exploited rather than to be mitigated for reliability in point-to-point communications.

There are several transmission schemes to perform packet scheduling in multiple antenna systems. It has been reported that assigning all transmit antennas to single user reduces channel randomness, and consequently is harmful to wireless packet scheduling [2]. Thus, the multiuser spatial multiplexing scheme assigning multiple users to multiple transmit antennas simultaneously has been introduced to achieve multiplexing gain [3], [4]. When the number of receive antennas is larger than that of transmit antennas, spatial multiplexing with linear detection has been investigated in [5], [6]. Furthermore, throughput performance of spatial multiplexing has been known to be asymptotically good as that of dirty paper coding [4]. It is important to note that uncorrelated transmit antennas are assumed in these schemes. In practical situations, however, there are antenna correlations among transmit antennas. There have been several researches investigating the effect of channel correlation on multiuser diversity systems. In the case of a large number of transmit and receive antennas, asymptotic analysis was performed for the scheme assigning all transmit antennas to single user under exponentially correlated fading in [7]. Angle between subspaces of users’ channel matrices to characterize inter-user spatial correlation was considered as a criterion for spatial mode selection and user grouping for multiplexing in [8]. However, these schemes have not provide answers to the question that whether the multiuser spatial multiplexing scheme is still optimal in highly correlated channels and under which conditions it is still optimal. Thus, the main focus of this paper includes investigating the effect of antenna correlation on the asymptotic performance of multiuser diversity and to characterize the system operation conditions in which each transmission scheme for multiuser diversity outperforms other schemes under antenna correlation.

In this paper we analyze the throughput performance of multiuser packet scheduling with various multi-antenna transmission modes. We derive the cumulative distribution of receive SNR for each transmission mode considering transmit antenna correlation. We then apply a lemma on the maximum of a sequence of independent random variables in order to obtain the asymptotic throughput growth rate. The derived asymptotic throughputs is quite close to the simulation results even in the case of the moderate number of users. From this analysis, we demonstrate that the optimal transmission scheme depends on the degree of the antenna correlation and the operating SNR.

This paper is organized as follows: We first describe the system model and various multi-antenna transmission modes in Section II. Then we present the asymptotic throughput analysis for the case of two transmit and one receive antennas in Section III. In Section IV, we extend the analysis to the case of the general number of antennas.

II. SYSTEM MODEL

We consider a Gaussian downlink fading channel with $M$ transmit antennas and one receive antenna. Let $\mathbf{x}(n) \in \mathbb{C}^M$ be the transmit vector at time slot $n$ with $E[\mathbf{x}(n)]^2 = \gamma$ and let $\mathbf{h}_k(n) = [h_{k,1}(n)\ h_{k,2}(n)\ \cdots\ h_{k,M}(n)]^T \in \mathbb{C}^M$ be a channel gain vector for user $k$ modelled by a circularly-symmetric complex Gaussian random vector with each element of zero mean and unit variance. We assume the channel vector $\mathbf{h}_k(n)$ is independent for all $k$ and $n$ and the correlation matrix at the transmit side is given as $E[\mathbf{h}_k(n)\mathbf{h}_k(n)^\dagger] = \mathbf{R}_{tx}$, where $(\cdot)^\dagger$ denotes the conjugate transpose operation. The received signal of user $k$ is represented as

$$y_k(n) = \mathbf{h}_k(n)^\dagger\mathbf{x}(n) + \eta_k(n), \quad k = 1, 2, \cdots, K \quad (1)$$

Paper approved by N. Al-Dhahir, the Editor for Space-Time, OFDM and Equalization of the IEEE Communications Society. Manuscript received December 29, 2005; revised July 6, 2007 and September 27, 2007. This paper was presented in part at the 2005 IEEE International Symposium on Information Theory, Adelaide, Australia, September 2005. This work was supported by an INHA University Research Grant.

D. Park is with the School of Information and Communication Engineering, Inha University, Incheon, 402-751 Korea (e-mail: dpark@ieee.org).

S. Y. Park is with the School of Information Technology, Kangwon National University, Chuncheon, 200-710 Korea (e-mail: young@ieee.org).

Digital Object Identifier 10.1109/TCOMM.2008.050671.
where \( \eta_k(n) \) is an additive white Gaussian noise with zero mean and unit variance. As a result, the average SNR at every receiver is \( \gamma \). For notational convenience, we may omit the time index \( n \).

The fading channel vector can be expressed as [9]

\[
h_k = R_{tx}^{1/2} w_k,
\]

where \( w_k \in \mathcal{C}^M \) is a circularly-symmetric complex Gaussian random vector with each i.i.d. element of zero mean and unit variance. We assume that each user perfectly estimates the channel vector and feeds channel quality information (CQI) back to the base station in an error-free manner. If we feed back whole information of channel parameters (i.e., quantized versions of complex channel gains) to fully exploit multiple antenna gains, then we can maximize the throughput by adopting intelligent transmit preprocessing schemes such as dirty paper coding [10], decomposition technique [11], and zero-forcing beamforming [12]. However, in general, these preprocessing schemes rely on the exact information of complex channel gains, thus requiring a large amount of feedback information. For feedback issues in transmit preprocessing schemes, a threshold-based feedback scheme has been introduced to limit the sum feedback rate of all users [13] and several practical multiuser MIMO transmissions have been proposed to suppress the co-channel interference based on some channel state information in [14],[15]. In this paper, we only consider a simple antenna allocation with no precoding at the transmitter in order to reduce the amount of feedback information. The base station determines which users to be transmitted at the next time slot based on this feedback information and transmits packets to those selected users without any power controls. We can classify the multi-antenna transmission schemes for packet scheduling according to the user allocation methods.

One of the transmission schemes is transmit diversity mode (TD), in which all antennas are assigned to one user to achieve the \( M \)-th order diversity for each transmitted symbol [16], [2]. For example, when \( M = 2 \), Alamouti’s space-time block code provides a simple method to encode data streams maintaining orthogonality between two antennas [17]. When each user feeds back the norm of the channel vector (i.e., \( |h_k|^2 \)), the base station determines which user is selected for transmission to maximize the throughput as

\[
T_{TD} = E \left[ \max_{k=1,2,\ldots,K} \log_2 \left( 1 + \frac{\gamma}{M} |h_k|^2 \right) \right].
\]

In uncorrelated multiple antenna channels, it is well known that the multi-antenna scheduling gain of TD decreases with the number of transmit antennas due to the channel hardening effect [2].

The second transmission scheme is antenna selection mode (AS), in which only one antenna is assigned to one user at each time slot [16], [3]. Each user determines the preferred transmit antenna \( m^*(k) \) which yields highest channel gain

\[
|h_{k,m^*(k)}|^2 = \max_{m=1,2,\ldots,M} |h_{k,m}|^2.
\]

Then, user \( k \) feeds back \( |h_{k,m^*(k)}|^2 \) along with the preferred antenna index \( m^*(k) \) and the base station determines the user whose channel gain \( |h_{k,m^*(k)}|^2 \) is the largest and sends packets to that user through transmit antenna \( m^*(k) \) to maximize the throughput as

\[
T_{AS} = E \left[ \max_{k=1,2,\ldots,K} \log_2 \left( 1 + \gamma \max_{m=1,2,\ldots,M} |h_{k,m}|^2 \right) \right].
\]

The third transmission scheme is spatial multiplexing mode (SM), in which each antenna is assigned to a different user to achieve spatial multiplexing gain [3], [4]. Each user determines the preferred transmit antenna \( m^*(k) \) which yields highest signal to interference and noise ratio (SINR)

\[
m^*(k) = \arg \max_{m=1,2,\ldots,M} \text{SINR}_{k,m} = \arg \max_{m=1,2,\ldots,M} \left( \sum_{i \in k} |h_{k,i}|^2 + \sigma^2 \right) / \sigma^2 = \max_{m=1,2,\ldots,M} \text{SINR}_{k,m},
\]

where \( \text{SINR}_{k,m} = |h_{k,m}|^2 / \left( \sum_{i \in k} |h_{k,i}|^2 + \sigma^2 \right) \) and \( \sigma^2 \equiv M/\gamma \). Then, user \( k \) feeds back \( \text{SINR}_{k,m^*(k)} \) along with the preferred antenna index \( m^*(k) \) and the base station determines the user whose SINR is largest for each antenna \( m \) to maximize the throughput as

\[
T_{SM} = E \left[ \sum_{m=1}^{M} \max_{k \in \{k| m^*(k) = m\}} \log_2 \left( 1 + \text{SINR}_{k,m^*(k)} \right) \right].
\]

When there are no correlation between transmit antennas, it is known that SM outperforms both of TD and AS due to the spatial multiplexing gain proportional to \( M \) [3], [4].

Table I summarizes the required feedback information for each transmission scheme. We assume that SNR or SINR information is represented as \( Q \) bits. Since TD has no feedback information about a preferred antenna index, \( Q \) bits is required. However, AS and SM require additional \( \log_2 M \) bits to represent a preferred antenna index. For example, when \( M = 2 \), only one more bit is required for AS and SM compared to that of TD.

### III. THROUGHPUT ANALYSIS FOR TWO TRANSMIT AND ONE RECEIVE ANTENNAS

In this section, we focus on the case of two transmit antennas for closed form expression of asymptotic analysis. We extend the analysis to multiple antenna cases in the next sections. The correlation matrix is given by

\[
R_{tx} = \begin{bmatrix} \rho & \rho^* \\ \rho^* & 1 \end{bmatrix},
\]

where \( |\rho| \leq 1 \). For simple analysis, we assume \( \rho \) is real and \( 0 \leq \rho \leq 1 \). If we decompose \( R_{tx} \) by using eigen decomposition

\[
R_{tx} = U \Lambda U^*,
\]

where \( \Lambda = \text{diag}(\rho, 1 \pm \rho) \) and \( U \) is a unitary matrix consisting of corresponding eigenvectors, then we get \( R_{tx}^{1/2} = U \Lambda^{1/2} U^* \). In [18], the exact capacity distribution for single user correlated MIMO systems was presented, but multiple users were not considered.

Note that the throughput expressions in (3), (4), and (5) have a similar form of \( \max_k \alpha_k \), where \( \alpha_k \)'s are random variables.
For the asymptotic analysis, we therefore rely on the following lemma.

**Lemma 1:** [1] Let $x_1, x_2, \ldots, x_K$ be a sequence of independent and identically distributed (i.i.d) random variables with $f_X(x)$ and $F_X(x)$ as probability density function (pdf) and cumulative density function (cdf) satisfying $F_X(x)$ is twice differentiable. If $\lim_{x \to -\infty} (1 - F_X(x))/f_X(x) = c > 0$, then $\max_k x_k - a_K$ converges in distribution to a limiting random variable with cdf $\exp(-e^{-x}/c)$, where $a_K$ is given by $F_X(a_K) = 1 - 1/K$.

This lemma says that $\max_k x_k$ behaves like $a_K$, as $K \to \infty$, which is the asymptotic growth rate of throughput.

In this section, we first derive the cumulative distribution of $x_k$ for each transmission mode considering antenna correlation. Then we apply Lemma 1 to get the asymptotic throughput growth rate.

### A. Transmit Diversity Mode

From (2), the vector norm $|h_k|^2$ can be expressed as

$$
|h_k|^2 = w_k^* U A U^* w_k = v_k^* A v_k = (1 - \rho)|v_k,1|^2 + (1 + \rho)|v_k,2|^2,
$$

(7)

where $v_k = [v_{k,1} \ v_{k,2}]^T = U^* w_k$ has the identical distribution with $w_k$, because a circularly-symmetric complex Gaussian distributed random vector with each i.i.d. zero-mean element is invariant by multiplication of constant unitary matrices. Thus, $|v_k,1|^2$ and $|v_k,2|^2$ are independent exponential random variables with unit mean.

If $\rho = 0$, $|h_k|^2$ has a chi-square distribution of order 4

$$
F_{|h_k|^2}(x) = 1 - e^{-x}(1 + x)
$$

and, if $\rho = 1$, it has an exponential distribution with mean 2

$$
F_{|h_k|^2}(x) = 1 - e^{-x/2}.
$$

(9)

In [19], the cdf of $T = \sum_{i=1}^L \lambda_i e_i$ is given by

$$
F_T(x) = 1 - \frac{1}{\Pi_{j=1}^L \lambda_j} \sum_{j=1}^L \lambda_j \exp\left(-\frac{x}{\lambda_j}\right) \prod_{k=1,k\neq j}^L \left(\frac{1}{\lambda_k} - \frac{1}{\lambda_j}\right),
$$

(10)

where $\lambda_j > 0$ are all distinct and $e_i$ are independent exponentially distributed with unit mean. Consequently, if $0 < \rho < 1$, the distribution of $|h_k|^2$ becomes

$$
F_{|h_k|^2}(x) = 1 - \frac{1 + \rho}{2\rho} e^{-\frac{x}{\rho}} + \frac{1 - \rho}{2\rho} e^{-\frac{x}{1-\rho}}.
$$

(11)

From (8), (9), and (11), $(1 - F_{|h_k|^2}(x))/f_{|h_k|^2}(x) \to 1 + \rho > 0$ as $x \to \infty$ for $0 \leq \rho \leq 1$, and consequently $\max_k |h_k|^2$ grows like $a_K$ according to Lemma 1, where $F_{|h_k|^2}(a_K) = 1 - 1/K$.

We can obtain $a_K \sim \log K + \log(1 + \log K)$ from (8) if $\rho = 0$, and $a_K \sim \log K$ from (9) if $\rho = 1$. Now, we consider the case of $0 < \rho < 1$. If $\rho$ is not too small, then the second term in the right hand side of (11) is dominated by the third term and we obtain $a_K \sim (1 + \rho) \log(1 + 2\rho K)$. However, if $\rho$ is very small and near 0, then both of the second and the third terms tend to infinity and (11) converges to (8). So, in the case of very small $\rho$, we use (8) rather than (11) for simple analysis.

Consequently, $T_{TD}$ in (3) grows like

$$
T_{TD} \sim \begin{cases} 
\frac{\log_2(1 + \frac{\rho}{2})}{2\rho} \log(1 + \log K), & \rho \approx 0, \\
\log_2 \left(1 + \frac{1 + \rho}{2\rho} \gamma \log(1 + \rho K)\right), & \rho \approx 1 
\end{cases}
$$

(12)

Note that (12) is the asymptotic throughput performance behavior for a given value of $\rho$ when the number of users increases. In point-to-point communications, it is known that the high correlation degrades performance because correlation reduces the diversity gain [19], [20]. However, according to (12), the high correlation improves the throughput performance of TD. This inconsistency can be explained as follows:

Combining signals from multiple transmit antennas that experience independent fading reduces signal fluctuation levels. The probability that the combined signal drops below a certain threshold is significantly lower than that of the received signal from single transmit antenna. So, this combining improves the performance of point-to-point communications. On the other hand, the probability that the combined signal exceeds a certain threshold is also significantly lower. It results in reduction of the multiuser diversity gain in point-to-multipoint communications. It is called channel hardening effects [2]. When the transmit antenna correlation is considered, it is harmful to the point-to-point cases, but it is beneficial for point-to-multipoint cases. The reason is that this correlation reduces the diversity effect of combining the signals. In conclusion, highly correlated multiple antennas are effectively transformed into single antenna, which mitigates the undesired channel hardening effects of the MIMO scheduling.

### B. Antenna Selection Mode

Since AS transmits the signal using only one transmit antenna, the correlated fading affects the performance slightly. For example, two extreme cases, $\rho = 0$ and $\rho = 1$, show slightly different performance. In this subsection, we investigate how the intermediate values of $\rho$ affect the performance even though the two extreme cases do not differ much.

In (2), $h_{k,1}$ and $h_{k,2}$ can be rewritten as $h_{k,1} = \sqrt{\rho} w_{k,0} + \sqrt{1 - \rho} w_{k,1}$ and $h_{k,2} = \sqrt{\rho} w_{k,0} + \sqrt{1 - \rho} w_{k,2}$ without loss of generality, where $w_{k,i}$'s are independent zero-mean complex Gaussian random variables with unit variance. If $\rho = 1$, the selection combining output $S = \max(|h_{k,1}|^2, |h_{k,2}|^2)$ has the exponential distribution $F_S(x) = 1 - e^{-x}$ and the asymptotic growth rate $a_K$ becomes $\log K$. If $\rho = 0$, $S$ has the distribution $F_S(x) = (1 - e^{-x})^2$ and $a_K = \log 2K$.

More generally, for $0 \leq \rho < 1$, $S$ has the following distribution [20]

$$
F_S(x) = 1 - 2e^{-x}Q\left(\sqrt{\frac{2x}{1 - \rho^2}}, \rho \sqrt{\frac{2x}{1 - \rho^2}}\right) + e^{-\frac{2x}{1 - \rho^2}} I_0\left(\frac{2\rho x}{1 - \rho^2}\right),
$$

(13)

where $Q(p, q)$ and $I_0(x)$ denote the Marcum-Q function and the zeroth-order modified Bessel function of the first kind. It is not tractable to find an analytic solution of $F_S(x) = 1 - 1/K$, so we instead use bounding techniques to get the asymptotic growth rate. If we apply the inequalities $Q(p, q) \leq 1$ and $I_0(x) \geq e^{x} I_0(b)/e^b$ for $0 \leq x \leq b$, where $b$ is an arbitrary
positive constant [21], then we get a lower bound of $F_S(x)$ as
\[
F_S(x) \geq 1 - 2e^{-x} + e^{-2x/(1+\rho)} I_0(b) \frac{I_0(b)}{eb} \equiv F_{S_1}(x)
\]
for $0 < x < b$. From $F_{S_1}(a_K) = 1 - 1/K$, we obtain the asymptotic growth rate
\[
a_K \sim \log 2K + \log \left( 1 - \frac{1}{2} (2K)^{-\frac{1}{x+2}} I_0(b) \frac{I_0(b)}{eb} \right).
\]
Without loss of generality we may set $b \equiv \log 2K$ because $a_K$ in (15) does not exceed $\log 2K$. Then, the asymptotic throughput $T_{AS}$ in (4) grows like
\[
T_{AS} \sim \log_2 \left( 1 + \gamma \log 2K \left( 1 - (2K)^{-\frac{1}{x+2}} I_0(b) \frac{I_0(b)}{eb} \right) \right)
\]
for $0 \leq \rho < 1$. We may also get a similar growth rate using an upper bound of $F_S(x)$. For large $K$, $T_{AS}$ grows like $\log_2(1 + \gamma \log 2K)$ irrespective of $\rho$. This implies that AS is robust to the antenna correlation when the number of users is large, because AS selects only one antenna without combining correlated signals from transmit antennas.

C. Spatial Multiplexing Mode

In our feedback model, each user feeds back the highest SINR and the corresponding antenna index. Let us compare this with another scheme such that each user feeds back not only the highest SINR but also the other SINR values and the base station selects the user whose SINR is largest for each antenna. Then, the throughput of our model in (5) can be upper-bounded by
\[
T_{SM} \leq E \left[ \sum_{m=1}^{2} \log_2 \left( 1 + \max_{k=1,2,\ldots,K} \text{SINR}_{k,m} \right) \right],
\]
(17)
where the right hand side denotes the throughput under the condition that all SINR values of each user are fed back to the base station. The equality may hold in (17) when each user is the maximum SINR user for at most one transmit antenna [4]. As in the random beamforming case [4], the probability that the same user has the maximum SINR value in two antennas at the same time is given by $1 - \Pr(\text{SINR}_{1,m} < 1)^K$, which is very small for large $K$. Consequently, it is likely that (17) represents the throughput of SM transmission mode.

If we define $Z_{k,m} \equiv 1 + \text{SINR}_{k,m}$, then $Z_{k,1}$ can be expressed as
\[
Z_{k,1} = \frac{|h_{k,1}|^2 + |h_{k,2}|^2 + \sigma^2}{|h_{k,2}|^2 + \sigma^2} w_k^* U A U^* w_k + \sigma^2
\]
\[
= w_k^* U A^{1/2} U^* w_k + \sigma^2
\]
\[
\text{for } \sigma^2 \equiv 2/\gamma, \quad v_k \equiv U^* w_k \quad \text{and} \quad B_1 = \sigma^2, \quad V_k = \sigma^2.
\]
\[
B_1 = \Lambda^{1/2} U^* \left[ \begin{array}{cc} 0 & 0 \\ 0 & 1 \end{array} \right] \Lambda^{1/2}
\]
\[
\text{where } \sigma^2 \equiv 2/\gamma, \quad v_k \equiv U^* w_k \quad \text{and} \quad B_1 = \Lambda^{1/2} U^* \left[ \begin{array}{cc} 0 & 0 \\ 0 & 1 \end{array} \right] \Lambda^{1/2}
\]
\[
= \frac{1}{2} \left[ 1 - \rho \sqrt{1 - \rho^2} \right].
\]
(18)
(19)
We can also express $Z_{k,2} = (v_k^* A v_k + \sigma^2)/(v_k^* B_2 v_k + \sigma^2)$, where $B_2$ is similarly defined. Now, we evaluate the cdf of $Z_{k,1}$ given as
\[
\Pr(Z_{k,1} \leq x) = \Pr \left( v_k^* (A - x B_1) v_k \leq \sigma^2 (x - 1) \right)
\]
for $x \geq 1$. Since $A - x B_1$ has two eigenvalues $(2 - x \pm \sqrt{4\rho^2 - 4\rho^2 x + x^2})/2$, we can express $v_k^* (A - x B_1) v_k = \lambda_1 \omega_1 - \lambda_2 \omega_2$, where $\lambda_1 = (2 - x + \sqrt{4\rho^2 - 4\rho^2 x + x^2})/2$ and $\lambda_2 = (2 - x - \sqrt{4\rho^2 - 4\rho^2 x + x^2})/2$ and $\omega_1$’s are independent exponentially distributed with unit mean. Consequently, the cdf of $Z_{k,1}$ becomes
\[
F_{Z_{k,1}}(x) = \Pr \left( \omega_1 \leq \frac{\lambda_2 \omega_2 + \sigma^2 (x - 1)}{\lambda_1} \right)
\]
\[
= \int_0^\infty (1 - e^{-(\lambda_2 + \sigma^2 (x - 1)/\lambda_1)}) e^{-t} dt
\]
\[
= 1 - \frac{\lambda_1}{\lambda_1 + \lambda_2} e^{-\sigma^2 (x - 1)/\lambda_1}
\]
\[
= 1 - \frac{2 - x + \sqrt{4\rho^2 - 4\rho^2 x + x^2}}{2\sqrt{4\rho^2 - 4\rho^2 x + x^2}} e^{-\frac{\sigma^2 (x - 1)}{2\sqrt{4\rho^2 - 4\rho^2 x + x^2}}}
\]
\[
= 1 - e^{-\frac{\sigma^2 (x - 1)}{2\sqrt{4\rho^2 - 4\rho^2 x + x^2}}}, \quad 1 \leq x < 2,
\]
\[
= 1, \quad x \geq 2,
\]
(20)
which implies that, as $K \to \infty$, $Z_{k,1}$ converges to 2. With the same reason, $Z_{k,2}$ also converges to 2. It means that $T_{SM}$ converges to 2 and there is no multiuser diversity gain when the antenna correlation is 1.

More generally, for $0 \leq \rho < 1$, the cdf is asymptotically expressed as
\[
F_{Z_{k,1}}(x) = 1 - \frac{1}{2} \left( 1 - \frac{x - 2}{\sqrt{4\rho^2 - 4\rho^2 x + x^2}} \right)
\]
\[
\cdot e^{-\frac{\sigma^2 (x - 2 + \sqrt{4\rho^2 - 4\rho^2 x + x^2})}{2(1-\rho^2)}}
\]
\[
= 1 - \frac{1 - \rho^2}{x - 2\rho^2} - O \left( \frac{1}{x^2} \right) \quad e^{-\frac{\sigma^2 (x - 1 + \rho^2)}{1-\rho^2}} + O \left( \frac{e^{-x}}{x^2} \right).
\]
\[
\text{In order to get the asymptotic growth rate of } Z_{k,1} \text{ such that } F_{Z_{k,1}}(a_K) = 1 - 1/K, \text{ we let } y_K = \frac{\sigma^2 (a_K - (1 + \rho^2))}{1-\rho^2}. \text{ Then, we have}
\]
\[
\frac{1}{1 + y_K/\sigma^2} e^{-y_K} - O \left( e^{-y_K/\sigma^2} \right) = 1.
\]
(21)
For large $K$, we get the solution $y_K$ is asymptotically

$$y_K = \log K - \log(1 + \log K + O(\log \log K)).$$

and consequently the asymptotic growth rate of $Z_{k,1}$ becomes

$$Z_{k,1} \sim 1 + \rho^2 + \frac{1 - \rho^2}{\sigma^2} \left( \log K - \log(1 + \log K/\sigma^2) \right).$$

As $Z_{k,2}$ also has the same growth rate, $T_{SM}$ in (5) grows like

$$T_{SM} \sim 2 \log_2 \left( 1 + \rho^2 + \frac{\gamma}{2} \left( 1 - \rho^2 \right) \left( \log K - \log \left( 1 + \frac{\gamma}{2} \log K \right) \right) \right).$$

Since this is decreasing with $\rho$, the antenna correlation is harmful to the throughput of the spatial multiplexing mode.

Fig. 1. Comparison of throughputs when (a) $\rho = 0.1$ and (b) $\rho = 0.9$.

Table II summarizes asymptotic throughputs of various multi-antenna scheduling schemes. For low correlation, SM outperforms both of TD and AS in terms of throughputs. While SM achieves the full multiplexing gains, TD and AS have no multiplexing gains because they transmit only one independent data stream. In addition, AS outperforms TD. The reason is that the transmit power is not split between two antennas and AS may be interpreted as if there were two times more competing users.

On the other hand, TD and AS outperforms SM when the antenna correlation is high. As high correlation reduces the effective rank of overall channels, transmitting more than one stream is not optimal. At $\rho = 1$, TD and AS become identical, because the two channels from two transmit antennas become identical.

D. Comparison

Table II summarizes asymptotic throughputs of various multi-antenna scheduling schemes. For low correlation, SM outperforms both of TD and AS in terms of throughputs. While SM achieves the full multiplexing gains, TD and AS have no multiplexing gains because they transmit only one independent data stream. In addition, AS outperforms TD. The reason is that the transmit power is not split between two antennas and AS may be interpreted as if there were two times more competing users.

On the other hand, TD and AS outperforms SM when the antenna correlation is high. As high correlation reduces the effective rank of overall channels, transmitting more than one stream is not optimal. At $\rho = 1$, TD and AS become identical, because the two channels from two transmit antennas become identical.

E. Numerical Results

We performed Monte Carlo simulations to illustrate the correlation effects on the multi-antenna scheduling. In the simulation, we employed two transmit antennas in the base station and one receive antenna in each mobile station. Moreover, we assumed that the average SNR $\gamma$ is 4 dB and the channel is Rayleigh fading with the correlation matrix $R_{tx}$ as in (6).

Fig. 1 (a) and (b) show the average throughputs with respect to the number of users at $\rho = 0.1$ and $\rho = 0.9$, respectively. We observed that the throughput increases as the number of users increases and the throughput predicted by the asymptotic analysis has the same slope with the simulation results regardless of the value of $\rho$. As expected, SM outperforms AS and TD, and has a larger slope for low correlation. In the case of high correlation, AS outperforms others.

Fig. 2 shows the average throughputs with respect to the degree of the correlation when there are 150 users. We observed that TD and AS performances are maintained even at high correlation, while SM performance is severely degraded. As discussed in section III-A and C, highly correlated antennas
of TD mitigates the undesired channel hardening effects, while that of SM degrades its gain. On the other hand, AS is robust to the effect of the correlation. Moreover, AS outperforms TD for all ranges of \( \rho \) as expected in Table II. It implies that SM known as the asymptotically optimal scheme in uncorrelated multiple antenna channels is no longer the optimal transmission scheme in highly correlated channels. Therefore, we conclude that the optimal scheduling changes depending on the degree of antenna correlation.

We may conceive that if random beams were employed, the performance degradation of SM could be mitigated. To investigate this effect, we performed a numerical simulation employing random beamforming in TD and AS [1], and multiple random beams in SM [4]. As illustrated in Fig. 3, even though there are throughput gains in SM due to increased randomness by using random beams compared to Fig. 2, AS still outperforms SM in highly correlated antenna channels. From this investigation, we confirm that SM employing the random beams cannot overcome the performance degradation resulting from high antenna correlations.

**IV. THROUGHPUT FOR M TRANSMIT AND N RECEIVE ANTENNAS**

In this section, we investigate more general case of \( M \) transmit and \( N \) receive antennas. We assume that the number of receive antennas \( N \) is greater than the transmit antennas \( M \) in order to apply a simple linear receiver. The received signal of user \( k \) is represented as

\[
y_k = H_k x + \eta_k, \quad k = 1, 2, \ldots, K,
\]

where \( x \in \mathbb{C}^M \) denotes the transmit vector with \( E|\text{ }x|^2 = \gamma \), \( H_k \in \mathbb{C}^{N \times M} \) a complex channel matrix, and \( \eta_k \in \mathbb{C}^N \) an additive white Gaussian noise with each element of zero mean and unit variance. The channel is assumed to be so highly scattered around receive antennas that the receive correlation is negligible [9]. So, the channel can be expressed as \( H_k = H_k^{(w)} R_{xz}^{1/2} \), where each element of \( H_k^{(w)} \) is zero-mean white Gaussian with unit variance.

In the last section, we have showed that AS outperforms TD for all antenna correlations. So, we omit the analysis for TD system for the following subsection. It has been known that there are many detection techniques for spatial multiplexing, such as zero-forcing (ZF), minimum mean square error (MMSE), ordered successive user cancellation (OSUC), and maximum-likelihood (ML) detections in [23]. Among them, we focus on spatial multiplexing with zero forcing (SM-ZF). SM-ZF achieves the same asymptotic with the optimal dirty paper coding under uncorrelated channels [5][6].

AS mode for the general number of transmit and receive antennas is very similar to that of two transmit and one receive antennas in Section III. We consider the maximal ratio combining of received signals to obtain array gain. User \( k \) measures the SNR per each transmit antenna, \( \text{SNR}_k,m = \gamma |H_k|_m^2, \quad m = 1, 2, \ldots, M \), where \( |H_k|_m \) denotes the \( m \)th column of the matrix \( H_k \), and determines the preferred transmit antenna \( m^*(k) \) which yield the highest SNR

\[
m^*(k) = \arg \max_{m=1,2,\ldots,M} \text{SNR}_{k,m}.
\]

Then, it feeds back \( \text{SNR}_{k,m^*(k)} \) with the preferred transmit antenna index \( m^*(k) \). The base station determines the user whose SNR \( \text{SNR}_{k,m^*(k)} \) is the largest and sends packet to that user through the transmit antenna \( m^*(k) \). The throughput of AS is given as

\[
T_{AS} = \log_2 \left( 1 + \gamma \max_{k=1,2,\ldots,K} \max_{m=1,2,\ldots,M} |H_k|_m^2 \right). \tag{29}
\]

In SM-ZF mode, each transmit antenna is assigned to a different user to achieve spatial multiplexing gain. User \( k \) detects the transmitted data through a ZF receiver

\[
H_k^\dagger y_k = x + H_k^\dagger \eta_k, \quad k = 1, 2, \ldots, K, \tag{30}
\]

where \( H_k^\dagger \) is the pseudo-inverse of \( H_k \). The post-processing SNR of the \( m \)th stream \( \Gamma_{k,m} \) can be expressed as

\[
\Gamma_{k,m} = \frac{\gamma}{M \sigma_m} \gamma_{k,m}, \tag{31}
\]

where \( \sigma_m \) is the \( (m,m) \)th element of \( R_{xz}^{-1} \) and \( \gamma_{k,m} \) has a central Chi-square distribution of order \( 2(M + 1) \) [24]. User \( k \) calculates the post-processing SNR \( \Gamma_{k,m} \) and determines the preferred transmit antenna \( m^*(k) \) which yields the highest post-processing SNR. Then, user \( k \) feeds back \( \Gamma_{k,m^*(k)} \) along with the preferred antenna index \( m^*(k) \) and the base station determines the user whose SNR is largest for each antenna \( m \) to maximize the throughput as

\[
T_{SM-ZF} = \sum_{m=1}^M \max_{k \in \{m^*(k) = m\}} \log_2 \left( 1 + \Gamma_{k,m^*(k)} \right). \tag{32}
\]

We performed Monte Carlo simulations to illustrate the antenna correlation effects in the case of 4 transmit and 4 receive antennas. We assumed that the average SNR \( \gamma \) is 6 dB and the channel is Rayleigh fading with the correlation matrix

\[
R_{xz} = \begin{bmatrix} 1 & \rho & \rho & \cdots & \rho \\ \rho & 1 & \rho & \cdots & \rho \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ \rho & \rho & \cdots & 1 & \rho \\ \rho & \rho & \cdots & \rho & 1 \end{bmatrix}. \tag{33}
\]
correlation matrices. The first correlation matrix is

\[
R_{t_x} = \begin{bmatrix}
1 & -0.3043 & 0.2203 & -0.1812 \\
-0.3043 & 1 & -0.3043 & 0.2203 \\
0.2203 & -0.3043 & 1 & -0.3043 \\
-0.1812 & 0.2203 & -0.3043 & 1 \\
\end{bmatrix},
\tag{34}
\]

which is used to model low antenna correlation in a pico-cell base station [25], and the second one is

\[
R_{t_x} = \begin{bmatrix}
1 & 0.76e^{j0.17\pi} & 0.43e^{j0.35\pi} & 0.25e^{j0.53\pi} \\
0.76e^{-j0.17\pi} & 1 & 0.76e^{j0.17\pi} & 0.43e^{j0.35\pi} \\
0.43e^{-j0.35\pi} & 0.76e^{-j0.17\pi} & 1 & 0.76e^{j0.17\pi} \\
0.25e^{-j0.53\pi} & 0.43e^{-j0.35\pi} & 0.76e^{-j0.17\pi} & 1 \\
\end{bmatrix},
\tag{35}
\]

which is used to model high antenna correlation in a micro-cell base station [25]. We varied SNR to investigate the effect of SNR on the throughput for each transmission mode. Figs. 5 (a) and (b) depict the average total throughputs with correlation matrix (34) and (35), respectively, where all 20 users have the same SNR as indicated in abscissa. In the figures, the hybrid mode combines AS and SM-ZF such that it determines the better transmission mode based on instantaneous channel realizations and transmits packets using that transmission mode. To implement this mode, we require both feedback information for AS and SM-ZF. We can observe that SM-ZF outperforms AS if the operating SNR is higher than 2 dB in Fig. 5 (a) and 11 dB in Fig. 5 (b); otherwise AS slightly outperforms SM-ZF. The reason can be explained as follows: In the parallel Gaussian channels, water pouring is required to achieve the maximum capacity. Allocating equal power to all channels approaches optimal as SNR increases, while allocating all power to one channel approaches optimal as SNR decreases [26]. In our context, AS transmits signal through only one transmit antenna, that is, allocates all power to one antenna, while SM-ZF allocates equal power to all antennas. So, the condition that SNR is below the threshold corresponds to the case that the power is not sufficient to allocate into all parallel channels and there are some parallel channels where the bottom of the vessel is above the water level and no power is allocated to them [26]. In summary, for given antenna correlation, there is an SNR threshold such that SM-ZF outperforms AS if SNR exceeds that threshold, and AS outperforms SM-ZF otherwise.

We also consider the case that each user sees various transmit correlation. In the simulation based on the spatial channel model in [27], the adjacent antenna distance was set to \(\lambda/2\), where \(\lambda\) is the wavelength, and the angular spread was set to \(25^\circ\) and the number of scatterers was 10. Fig. 6 depicts the throughput with respect to SNR when the number
of users is 20. In this case, the SNR threshold is 12 dB to determine which transmission mode is better. So, we can apply this analysis for the cases of various transmit correlations.

For practical systems, it is necessary to consider the amount of channel feedback information. If we employ simple switching between the two transmission modes considering the long-term information such as the operating SNR, the number of users, and the antenna correlation, then we get the performance similar to the hybrid mode. So we can reduce the amount of feedback with negligible performance degradation.

To investigate the effect of heterogeneous user SNR, we employed the proportional fair scheduling [1] for 20 users whose SNR are equally spaced from -2 dB to 7.5 dB. Figs. 7 (a) and (b) show average throughput of each heterogeneous SNR user with correlation matrix (34) and (35), respectively. In Fig. 7 (a), SM-ZF outperforms AS if the operating SNR is higher than 2 dB, while in Fig. 7 (b), AS outperforms SM-ZF for all SNR range. It matches our expectation, since the SNR threshold for correlation matrix (34) is 2 dB in Fig. 5 (a) and that for correlation matrix (35) is 11 dB in Fig. 5 (b).

V. CONCLUSION

Among several multiuser transmission schemes, SM has been known as the asymptotically optimal scheme in uncorrelated multiple antenna channels [4]. In the case of correlated channels, we investigated whether this is still valid argument through asymptotic analysis. We found that there is an SNR threshold such that SM outperforms other schemes if SNR exceeds the threshold. Additionally, high antenna correlation makes the threshold high. This is because the spatial multiplexing gain cannot be fully exploited for high correlation. Therefore, it is concluded that the optimal transmission scheme depends on the degree of the correlation and the operating SNR.

We have considered only SM-ZF as the spatial multiplexing scheme. However, there are other spatial multiplexing schemes exploiting multiplexing gain. Therefore, it is for further study to investigate the performance of these schemes.

ACKNOWLEDGMENT

The authors appreciate the useful comments from the anonymous reviewers.

REFERENCES


[14] 3GPP TSG RAN WG1, Draft Report of 3GPP TSG RAN WG1 #49bis v0.1.0, June 2007.


Daeyeong Park (S’00–M’05) received the B.S. and M.E. degrees in electrical engineering and the Ph.D. degree in electrical engineering and computer science, all from Seoul National University, Seoul, Korea, in 1998, 2000, and 2004, respectively. He was with Samsung Electronics as a Senior Engineer from 2004 to 2007, contributing to the development of next-generation wireless systems based on the MIMO-OFDM technology. From 2007 to 2008, he was with the University of Southern California, Los Angeles, CA, as a Postdoctoral Researcher. In March 2008, he joined the faculty of the School of Information and Communication Engineering, Inha University, Korea. His research interests include communication systems, wireless networks, multiuser information theory, and resource allocation.

Seung Young Park (S’97–M’03) received the B.S., M.S., and Ph.D. degrees in electrical engineering from Korea University, Seoul, Korea, in 1997, 1999, and 2002, respectively. From April 2003 to December 2005, he was with Samsung Advanced Institute of Technology, Kiheung, Korea, where he was a Senior Engineer, working on several projects in the field of next-generation wireless mobile communications. From January 2006 to February 2007, he was with the Department of Electrical and Computer Engineering, Purdue University, West Lafayette, IN, where he was a Postdoctoral Research Associate. Since March 2007, he has been with the School of Information Technology, Kangwon National University, Chuncheon, Korea, where he is an Assistant Professor. His research interests include iterative detection, multicarrier systems, multiuser communications, and radio resource management.